Change of Job and Change of Residence – Geographical Mobility of the Labour Force

Deding Mette & Filges Trine

Globalisation, society and welfare
Working Paper 9:2004

Working Paper

Socialforskningsinstituttet
The Danish National Institute of Social Research

Change of Job and Change of Residence – Geographical Mobility of the Labour Force

Deding Mette & Filges Trine

Globalisation, society and welfare
Working Paper 9:2004

Change of Job and Change of Residence - Geographical Mobility of the Labour Force*

Mette Deding[†] and Trine Filges[‡]

December 21, 2004

Abstract

Solving regional labour market discrepancies through geographical mobility has gained increased political interest. The decision of changing job is closely related to the decision of changing residence as either change may imply a change in commuting cost. In this paper we set up a search model that can explain the residence and job changing behaviour of workers. The model is a double search model, in the sense that workers search for better jobs and dwellings simultaneously. Results show that the interrelationship between change of job and change of residence is very complex. However, scope for mobility inducing policies are especially found through the housing market parameters.

Keywords: job, residence, mobility, search

JEL classifications: J61, J63, J68

---

*We want to thank participants at the EEA conference, Venice, Italy, 2002, at the EALE conference, Paris, France, 2002, at the workshop in Geographical Mobility, SFI, 2004 and Jos van Ommeren.

†Herluf Trolles Gade 11, DK-1052 Copenhagen K Phone: +45 33 48 08 85, Fax: +45 33 48 08 33, E-mail: mcd@sfi.dk

‡Herluf Trolles Gade 11, DK-1052 Copenhagen K Phone: +45 33 48 09 26, Fax: +45 33 48 08 33, E-mail: tif@sfi.dk

1. Introduction

In many countries, the labour market is characterized by large regional discrepancies. For instance in Denmark, during the period 1995-2000 the highest regional unemployment rate was twice as high as the lowest regional unemployment rate (measured at county level). Furthermore, the discrepancies are permanent. The ordering of regional unemployment rates is thus preserved over the business cycle, with minor exceptions. Consequently, from a policy point of view adjustment of the labour market through geographical mobility is interesting. The question is whether it is possible to induce labour to migrate from regions having above average unemployment to regions having below average unemployment.

The decision of changing job is closely related to the decision of changing residence. Job location and residence location are connected by commuting distance. If an individual changes to a job further away, the commuting distance increases. And if the increase is large, the individual may consider to change residence. The individual however, takes this into account when deciding whether to take the new job.

In Denmark, the general point of view is that whereas job mobility is rather high (high turnover rates), residential mobility is relatively low. In order to understand this, we investigate what a theoretical model predicts about the relation between job and residence mobility.

The aim of the paper is thus to set up a model that explains the job and residence changing behaviour of individuals. The model is a double search model, in the sense that individuals search for better jobs and better residences simultaneously. The labour market is characterized by non-overlapping regional commuting areas, which is an appropriate description of the Danish labour market (see Andersen, 2000). A regional commuting area is defined as an area where the in and out commuting is below a certain value. Individuals thus search for better jobs both in their current region of residence (home) and in another region (abroad). At the same time, the individuals search for better residences in the home region, but if they change to a job abroad they also search for a residence abroad. Based on this framework, it is possible to investigate how job change decisions and residence change decisions are related. The model is inspired by van Ommeren et al. (1999), although in that model, urban areas are overlapping.

The paper is organised as follows. In the next section the theoretical model of job location and residence location is introduced, followed by examination of the optimal strategy based on the model in section 3. In section 4, we discuss the policy implications of the model, with special reference to the structure of the Danish labour market. Finally, concluding remarks are found in section 5.

2. A model of job location and residence location

The theoretical model describes the relationship between job location and residence location from a search perspective. Search theory is relevant since it is based on the idea of individuals moving between different states and thus fits the dynamic structure of the problem. Furthermore, uncertainty is explicitly treated in this framework.

Individuals are assumed to search for better jobs and dwellings, maximizing the discounted future flow of job utility and place utility minus commuting costs, taking into account the costs of changing job or residence. Job offers and residence offers arrive at an exogenously specified rate, and these offers are then accepted or rejected instantly.

The geographical structure of the labour market is very important for the outcome of the model. Several studies of the Danish labour market have indicated that although Denmark is a small country, it is characterized by non-overlapping regional commuting areas (see for instance the Ministry of the Environment, 2001, or Andersen, 2000). By a regional commuting area is understood that people commute within the region, but only seldom between regions. Over time, these regions are getting larger, i.e. people are commuting longer distances, but in 2000 Denmark could still be divided into approximately 45 non-overlapping commuting regions (see the Ministry of the Environment, 2001).

Thus, there are two geographical distinct labour markets in the model: home, h, and abroad, a. The residence is characterized by a value, r, which measures specific dwelling characteristics, net of costs, as well as location characteristics. For convenience we refer to r as the place utility. The job is characterized by a value, w, which includes the wage as well as other non-pecuniary benefits associated with the job. For convenience we refer to w as the wage. Job and residence location are connected by the commuting distance, z. When job and residence are in the same region, z equals 0, otherwise z is set equal to 1. Utility, $u(.)$, derived in a small period of time, dt, is then $u(w,r,z)dt$. We assume that $u_w > 0$, $u_r > 0$, and $u_z < 0$, that is utility is increasing in wages and place utility, but decreasing in commuting distance. Furthermore we assume that the instantaneously utility function $u(w,r,z)$ can be divided into the utility of the wage and place utility and a cost function of commuting, i.e. $u(w,r,z) = u(w,r) - c(z)$. Utility is strictly concave in w and r and the cost function is zero if $z = 0$, otherwise it is strictly positive: $u_{ww} < 0$, $u_{rr} < 0$, $c(0) = 0$ and $c(1) > 0$.

Individuals are assumed to search both for better jobs and for better residences. Job offers in the home region arrive with the rate p_j^h. Individuals also search in the labour market abroad, where job offers arrive with the rate, p_j^a. Notice, that we do not put any restrictions on the job offer arrival rates, i.e. whether p_j^a is higher, equal to or less than p_j^h. Residence offers arrive at the rate p_r. The residence offer arrival rate is at the same magnitude, whether it comes from the home region or the region abroad.[1] However, we assume that individuals only find it relevant to search for a new residence in the region where they currently work. Hence, they only search for residence in the region abroad, in case they have accepted a job there. Otherwise the residence search is directed towards the home region. We thus assume, that moving regions is conditioned on having accepted a job in the region abroad. In the present model geographical mobility is thus always initiated by job mobility. Geographical mobility due to personal factors are, however, not ruled out. It just requires that an acceptable job is found in the region abroad. Consequently, the home region always measures the region of residence.

Job offers and residence offers arrive independently. Apart from few jobs with an official residence, this is a realistic assumption. Furthermore, the decision to accept or reject an offer must be taken instantly before other offers arrive, thus pooling of offers is not allowed.

[1] This assumption can be justified for example by the fact that all relevant information about a residence can be found at the internet, no matter in which part of the country you search. The job offer arrival rate, on the other side, may depend, both on how familiar the job searcher is with the region, but also on the type of jobs available in the region.

The individual takes into account the once and only costs of changing job and residence. The cost of changing job within the region is c_j^h, the cost of changing residence within the region is c_r^h, the cost of changing job between regions is c_j^a, and the cost of changing residence between regions is c_r^a. The cost of changing residence includes primarily moving costs, and it is natural to assume that these increase with distance, that is $c_r^a > c_r^h$. For convenience we write the cost of changing residence as a function of commuting: $c_r(z) = c_r^h + zc_r^a$ Job moving costs include for instance effort to get acquainted with the new job. However, this cost is probably not related to distance, therefore we assume $c_j^h = c_j^a = c_j$.

The model outlined here is based on individuals, i.e. the interrelationship between for instance spouses in a couple is not modelled. Evidently, individuals are not independent of the people they live with, whether this is a partner and/or children, and the decision to change residence is not taken independent of the partner's wishes. However, modelling the search behaviour of both partners in a couple is beyond the scope of this paper[2].

We now turn to the optimal strategy of the individuals in the model and examine the interrelationship between the decision to change job and the decision to change residence.

3. Optimal strategy

The individual's optimal strategies are determined on basis of their expected lifetime utilities, that are functions of present utility and the excepted gains from future job and residence offers. Given a wage offer the expectation is taken with respect to the distribution of wages, and given a residence offer the expectation is taken with respect to the distribution of residences, where the distribution variable in both cases is x. The wage is distributed between 0 and $\tilde{w}$, with density function $f(w)$, and distribution function $F(w)$, where $F(\tilde{w}) = 1$. The place utility is distributed between 0 and $\tilde{r}$, with density function $g(r)$, and distribution function $G(r)$, where $G(\tilde{r}) = 1$.

Let $V(w, r, z)$ denote the present-discounted value of the expected utility stream of an individual. If the individual is working and living within the same regional commuting

[2] See Ommeren et al (1998) for an analysis of moving behaviour of two-earner households.

area, z is equal to 0 and if the individual is living and working in separate regional areas, z is equal to 1.

Hence, $V(w,r,z)$ is determined by the equation:

$$\begin{aligned}\delta V(w,r,z) &= u(w,r)+c(z)+p_j^h E_{\max}\left[V(w,r,z),V(x,r,0)-c_j\right]+\\&\quad p_j^a E_{\max}\left[V(w,r,z),V(x,r,1)-c_j\right]+\\&\quad p_r E_{\max}\left[V(w,r,z),V(w,x,0)-c_r(z)\right]-\\&\quad \left(p_j^h+p_r+p_j^a\right)V(w,r,z)\end{aligned} \tag{3.1}$$

Note, that we do not consider the possibility of being involuntarily separated from a job, as it only complicates the analysis without any qualitative changes[3]. The difference between a situation where $z=0$ and $z=1$ is that in the latter, the worker has to take the commuting cost, $c(1)$, into account. Furthermore, the relevant residence offer is now in the new work region, i.e. the region abroad. This is due to the assumption, that job search is not spatially restricted, whereas search of a better residence is restricted to the regional commuting area where the individual works. Hence, only acceptance of a new job may increase the commuting distance, whereas acceptance of a new residence can only maintain or decrease the commuting distance. Furthermore, the commuting cost, $c(1)$, has to be thought of as 'unacceptably' high, implying that workers will never consider moving residence in the home region but rather they will start searching for a new residence in the new work region abroad. In case a job offer in the home region is accepted before an acceptable residence offer in the work region abroad arrives, the residence search will again be directed towards the home region.

The worker has to decide when a job or residence offer is acceptable. The optimal decision rules are given by four reservation wages and place utilities: s_j^h, the wage at which the worker is indifferent between accepting a job in the home region or continue working

[3] A constant separation probability, as is the usual assumption in search models, only ads a constant positive compensation term to the reservation values.

in the present job, s_j^a, the wage at which the worker is indifferent between accepting a job abroad or continue working in the present job, s_r^h, the place utility at which the worker is indifferent between moving to a new residence in the home region or continue living in the same residence, and finally s_r^a, is the place utility at which the worker is indifferent between moving to a residence abroad, or continue living in the same residence given that the worker is working abroad.

The reservation wage, $s_j^h(w,r,0)$, defines the minimum acceptable job offer in the home region for a worker, who do not commute and is currently earning w and having place utility r. This home-region reservation wage is found by taking the derivative of (3.1) with respect to $s_j^h(w,r,0)$, which gives:

$$-V\left(s_j^h,r,0\right)+V(w,r,0)+c_j=0 \tag{3.2}$$

Using equation (3.1) and rearranging, we obtain that $s_j^h(w,r,0)$ is the solution to:

$$\begin{aligned}
u\left(s_j^h,r\right) &= u(w,r)+\\
c_j\left(\delta+p_j^h\left(1-F\left(s_j^h\left(s_j^h,r,0\right)\right)\right)+p_j^a\left(1-F\left(s_j^a\left(s_j^h,r,0\right)\right)\right)\right)&+\\
p_j^h\int_{s_j^h(w,r,0)}^{s_j^h\left(s_j^h,r,0\right)}\left(V(x,r,0)-V\left(s_j^h,r,0\right)\right)f(x)\,dx&+\\
p_j^a\int_{s_j^a(w,r,0)}^{s_j^a\left(s_j^h,r,0\right)}\left(V(x,r,1)-V\left(s_j^h,r,0\right)\right)f(x)\,dx&+\\
p_r\int_{s_r^h(w,r,0)}^{\tilde{r}}\left(V(w,x,0)-c_r^h-V(w,r,0)\right)g(x)\,dx&-\\
p_r\int_{s_r^h\left(s_j^h,r,0\right)}^{\tilde{r}}\left(V\left(s_j^h,x,0\right)-c_r^h-V\left(s_j^h,r,0\right)\right)g(x)\,dx&
\end{aligned} \tag{3.3}$$

The terms in (3.3) are interpreted as follows:

1) The term $c_j\delta$ is the long run compensation of incurring the job moving cost, c_j.

2) The worker takes into account that after accepting a job offer in the home region he receives new job offers in the home region and abroad, at the rates p_j^h and p_j^a. The

terms $c_j p_j^h \left(1 - F\left(s_j^h\left(s_j^h, r, 0\right)\right)\right)$ and $c_j p_j^a \left(1 - F\left(s_j^a\left(s_j^h, r, 0\right)\right)\right)$ can be interpreted as the compensation for leaving the new job again, if a better job offer arrives.

However, the worker sets the reservation wage higher than would be necessary to be compensated for the job moving cost, only. The terms

$$p_j^h \int_{s_j^h(w,r,0)}^{s_j^h\left(s_j^h,r,0\right)} \left(V\left(x,r,0\right) - V\left(s_j^h, r, 0\right)\right) f\left(x\right) dx$$

and

$$p_j^a \int_{s_j^a(w,r,0)}^{s_j^a\left(s_j^h,r,0\right)} \left(V\left(x,r,1\right) - V\left(s_j^h, r, 0\right)\right) f\left(x\right) dx,$$

may be interpreted as "to guard against the possibility of getting another wage offer after changing job that would have been preferred before changing, but which is not sufficiently high to induce a second change" (see Hey and McKenna (1979) and Burgess (1992)).

3) Finally, the worker takes into account the future residence offers, and thereby the expectation of future gains due to moving residence. If the position at the housing market worsens, in the sense that the expected future gain due to moving residence is lower after having accepted a new job in the home region compared to the current housing market situation, the compensation must be positive, otherwise it is negative.

However, it can be shown that, as (by definition) the commuting cost is not influenced by accepting a new job in the home region, the future position at the housing market (in the home region) will not be affected. Consequently the terms

$$p_r \int_{s_r^h(w,r,0)}^{\tilde{r}} \left(V\left(w,x,0\right) - c_r^h - V\left(w,r,0\right)\right) g\left(x\right) dx$$

and

$$p_r \int_{s_r^h\left(s_j^h,r,0\right)}^{\tilde{r}} \left(V\left(s_j^h,x,0\right) - c_r^h - V\left(s_j^h,r,0\right)\right) g\left(x\right) dx$$

cancel out, as $s_r^h\left(w,r,0\right) = s_r^h\left(s_j^h,r,0\right)$.

Rewriting equation (3.3), the minimum acceptable wage offer in the home region becomes the solution to:

$$u\left(s_j^h, r\right) = u\left(w, r\right) + \tag{3.4}$$
$$c_j\left(\delta + p_j^h\left(1 - F\left(s_j^h\left(s_j^h, r, 0\right)\right)\right) + p_j^a\left(1 - F\left(s_j^a\left(s_j^h, r, 0\right)\right)\right)\right) +$$
$$p_j^h \int_{s_j^h(w,r,0)}^{s_j^h\left(s_j^h, r, 0\right)} \left(V\left(x, r, 0\right) - V\left(s_j^h, r, 0\right)\right) f\left(x\right) dx +$$
$$p_j^a \int_{s_j^a(w,r,0)}^{s_j^a\left(s_j^h, r, 0\right)} \left(V\left(x, r, 1\right) - V\left(s_j^h, r, 0\right)\right) f\left(x\right) dx +$$

The reservation wage, s_j^h, is seen to be unambiguously higher than the current wage, as all of the compensation terms are positive.

Turning to the second reservation wage, s_j^a, this gives the minimum acceptable job offer in the region abroad for a worker currently earning w and having place utility r. The reservation wage, $s_j^a\left(w, r, 0\right)$, is obtained by taking the derivative of (3.1) with respect to s_j^a, and hence, the reservation wage $s_j^a\left(w, r, 0\right)$, is obtained by solving:

$$-V\left(s_j^a, r, 1\right) + V\left(w, r, 0\right) + c_j = 0 \tag{3.5}$$

Using equation (3.1) and rearranging, $s_j^a\left(w, r, 0\right)$ is the solution to:

$$u\left(s_j^a, r\right) = u\left(w, r\right) + c\left(1\right) +$$
$$c_j\left(\delta + p_j^h\left(1 - F\left(s_j^h\left(s_j^a, r, 1\right)\right)\right) + p_j^a\left(1 - F\left(s_j^a\left(s_j^a, r, 1\right)\right)\right)\right) +$$
$$p_j^h \int_{s_j^h(w,r,0)}^{s_j^h\left(s_j^a, r, 1\right)} \left(V\left(x, r, 0\right) - V\left(s_j^a, r, 1\right)\right) f\left(x\right) dx + \tag{3.6}$$
$$p_j^a \int_{s_j^a(w,r,0)}^{s_j^a\left(s_j^a, r, 1\right)} \left(V\left(x, r, 1\right) - V\left(s_j^a, r, 1\right)\right) f\left(x\right) dx +$$
$$p_r \int_{s_r^h(w,r,0)}^{\tilde{r}} \left(V\left(w, x, 0\right) - c_r^h - V\left(s_j^a, r, 1\right) + c_j\right) g\left(x\right) dx -$$
$$p_r \int_{s_r^a\left(s_j^a, r, 1\right)}^{\tilde{r}} \left(V\left(s_j^a, x, 0\right) - \left(c_r^h + c_r^a\right) - V\left(s_j^a, r, 1\right)\right) g\left(x\right) dx$$

Equation (3.6) is equal to (3.3) regarding the job change terms. Since the cost of changing job is assumed to be the same, no matter whether the new job is in the home region or abroad, the compensation regarding new job offers is identical in the two cases. But contrary to (3.3), the worker in the present case requires a compensation due to the imposed commuting cost, $c(1)$. Furthermore, the last two terms in (3.6) are different from the last two terms in (3.3), because the residence situation is different. After the job change to the region abroad, the search of a new residence not only serves to increase place utility, but also to decrease commuting cost. Hence it may, very well, be the case that the position at the housing market improves in the sense that the expected future gain due to moving residence is higher after having accepted a job abroad. Thus, we can not rule out that the compensation regarding the housing situation may be negative. In fact it can be shown that the compensation *is* negative. It is shown in Appendix A that the reservation place utility after having changed job is lower than the reservation place utility if no job change takes place:

$$s_r^a\left(s_j^a, r, 1\right) < s_r^h\left(w, r, 0\right)$$

This implies that, the compensation regarding the housing situation is negative (see Appendix A):

$$\begin{aligned} p_r \int_{s_r^h(w,r,0)}^{\tilde{r}} \left(V\left(w, x, 0\right) - c_r^h - V\left(s_j^a, r, 1\right) + c_j\right) g\left(x\right) dx - \\ p_r \int_{s_r^a\left(s_j^a, r, 1\right)}^{\tilde{r}} \left(V\left(s_j^a, x, 0\right) - c_r^h - c_r^a - V\left(s_j^a, r, 1\right)\right) g\left(x\right) dx < 0 \end{aligned} \tag{3.7}$$

The negative housing compensation term implies that workers may accept a lower wage *net of commuting costs*, because the commuting costs may be reduced by moving to a residence abroad. However, it can be shown that (see Appendix A), not only is the reservation wage inducing a job change to the region abroad unambiguously higher than the current wage, but also that it is higher than the reservation wage inducing a job change in the home region:

$$s_j^a\left(w, r, 0\right) > s_j^h\left(w, r, 0\right) > w$$

An acceptable job offer thus has to give a higher utility if it arrives from the region abroad compared to the home region. The reason is the increased commuting cost, which can only be reduced by imposing yet another cost, either by moving residence or accepting a new job offer in the home region.

After the discussion of the two reservation wages, we turn to the reservation place utilities. The first reservation place utility, s_r^h, is the minimum acceptable place utility offer in the home region for a worker currently earning w and having place utility r. The reservation place utility, $s_r^h(w, r, 0)$, is found by taking the derivative of (3.1):

$$-V\left(w, s_r^h, 0\right) + V(w, r, 0) + c_r^h = 0 \tag{3.8}$$

Rewriting (3.8) using equation (3.1) $s_r^h(w, r, 0)$ is found to be the solution to:

$$\begin{aligned}
u\left(w, s_r^h\right) &= u(w, r) + \\
& c_r^h\left(\delta + p_r\left(1 - G\left(s_r^h\left(w, s_r^h, 0\right)\right)\right)\right) + \\
& p_r \int_{s_r^h(w,r,0)}^{s_r^h\left(w, s_r^h, 0\right)} \left(V(w, x, 0) - V\left(w, s_r^h, 0\right)\right) g(x)\, dx + \\
& p_j^h \int_{s_j^h(w,r,0)}^{\tilde{w}} \left(V(x, r, 0) - V(w, r, 0) - c_j\right) f(x)\, dx - \\
& p_j^h \int_{s_j^h\left(w, s_r^h, 0\right)}^{\tilde{w}} \left(V\left(x, s_r^h, 0\right) - V\left(w, s_r^h, 0\right) - c_j\right) f(x)\, dx + \\
& p_j^a \int_{s_j^a(w,r,0)}^{\tilde{w}} \left(V(x, r, 1) - V(w, r, 0) - c_j\right) f(x)\, dx - \\
& p_j^a \int_{s_j^a\left(w, s_r^h, 0\right)}^{\tilde{w}} \left(V\left(x, s_r^h, 1\right) - V\left(w, s_r^h, 0\right) - c_j\right) f(x)\, dx
\end{aligned} \tag{3.9}$$

The optimal reservation place utility inducing a residence move in the home region depends on the expectation of future job and residence offers, and the terms in (3.9) are interpreted as follows:

1) The term $c_r^h \delta$ is the long run compensation of incurring the residence moving cost, c_r^h.

2) The worker takes into account that after accepting a residence offer in the home region he receives other residence offers in the home region, at the rate p_r. The term

$c_r^h p_r \left(1 - G\left(s_r^h\left(w, s_r^h, 0\right)\right)\right)$ can be interpreted as the compensation for leaving the new residence again. However, parallel to the discussion of reservation wages, the worker sets the reservation place utility higher than would be necessary to be compensated for the residence moving cost. The term $p_r \int_{s_r^h(w,r,0)}^{s_r^h\left(w,s_r^h,0\right)} \left(V\left(w,x,0\right) - V\left(w,s_r^h,0\right)\right) g\left(x\right) dx$ can thus be interpreted as "the guard against the possibility of getting another residence offer after changing residence that would have been preferred before changing, but which is not sufficiently high to induce a second change".

3) Finally, the worker takes the future job offers into account and thereby the expected future gains of changing jobs. If the position at the job market, home or abroad, worsens, in the sense that the expected future gain due to changing job is lower after having accepted a new residence in the home region compared to the current job market situation, the compensation will be positive, otherwise it is negative. However, as (by definition) the commuting cost is not influenced by accepting a new residence in the home region, it can be shown that the future position at the job market in the home region is not affected by the residence move. Hence, the terms

$$p_j^h \int_{s_j^h(w,r,0)}^{\tilde{w}} \left(V\left(x,r,0\right) - c_j - V\left(w,r,0\right)\right) f\left(x\right) dx$$

and

$$p_j^h \int_{s_j^h\left(w,s_r^h,0\right)}^{\tilde{w}} \left(V\left(x,s_r^h,0\right) - c_j - V\left(w,s_r^h,0\right)\right) f\left(x\right) dx$$

cancel out, as $s_j^h\left(w,r,0\right) = s_j^h\left(w,s_r^h,0\right)$.

Somewhat surprising, the expected future situation in the job market abroad is also not affected by the change of residence. It can be shown that the change of residence do not imply that a higher wage offer is needed in order to accept a job offer from the region abroad, i.e. $s_j^a\left(w,s_r^h,0\right) = s_j^a\left(w,r,0\right)$. Therefore, the last two terms in (3.9)

$$p_j^a \int_{s_j^a(w,r,0)}^{\tilde{w}} \left(V\left(x,r,1\right) - V\left(w,r,0\right) - c_j\right) f\left(x\right) dx$$

and

$$p_j^a \int_{s_j^a\left(w,s_r^h,0\right)}^{\tilde{w}} \left(V\left(x,s_r^h,1\right) - V\left(w,s_r^h,0\right) - c_j\right) f\left(x\right) dx$$

cancel out.

Consequently, the minimum acceptable residence offer in the home region reduces to:

$$\begin{aligned} u\left(s_r^h,r\right) &= u\left(w,r\right) + \\ c_r^h &\left(\delta + p_r\left(1 - G\left(s_r^h\left(w,s_r^h,0\right)\right)\right)\right) + \\ p_r &\int_{s_r^h(w,r,0)}^{s_r^h\left(w,s_r^h,0\right)} \left(V\left(w,x,0\right) - V\left(w,s_r^h,0\right)\right) g\left(x\right) dx \end{aligned} \tag{3.10}$$

Parallel to the reservation wages, this reservation place utility is unambiguously higher than the current place utility. The worker thus requires a positive compensation in terms of lifetime utility in order to consider a change of residence.

The last strategy to consider is the second reservation place utility, s_r^a, which gives the minimum acceptable place utility offer in the region abroad (where the individual works) for a worker currently earning w, having place utility r and a commuting distance of $z = 1$. The reservation place utility, $s_r^a\left(w,r,1\right)$, is obtained by taking the derivative of (3.1) with respect to s_r^a, yielding:

$$-V\left(w,s_r^a,0\right) + V\left(w,r,1\right) + c_r^a + c_r^h = 0 \tag{3.11}$$

Inserting (3.1) we find that $s_r^a\left(w,r,1\right)$ is the solution to:

$$
\begin{aligned}
u\left(w,s_r^a\right) = u\left(w,r\right) - c\left(1\right) + \left(c_r^h + c_r^a\right)\delta + \\
p_r \int_{s_r^a(w,r,1)}^{\tilde{r}} \left(V\left(w,x,0\right) - V\left(w,s_r^a,0\right)\right) g\left(x\right) dx - \\
p_r \int_{s_r^h(w,s_r^a,0)}^{\tilde{r}} \left(V\left(w,x,0\right) - V\left(w,s_r^a,0\right) - c_r^h\right) g\left(x\right) dx + \\
p_j^h \int_{s_j^h(w,r,1)}^{\tilde{w}} \left(V\left(x,r,0\right) - V\left(w,r,1\right) - c_j\right) f\left(x\right) dx - \\
p_j^h \int_{s_j^h(w,s_r^a,0)}^{\tilde{w}} \left(V\left(x,s_r^a,0\right) - V\left(w,s_r^a,0\right) - c_j\right) f\left(x\right) dx + \\
p_j^a \int_{s_j^a(w,r,1)}^{\tilde{w}} \left(V\left(x,r,1\right) - V\left(w,r,1\right) - c_j\right) f\left(x\right) dx - \\
p_j^a \int_{s_j^a(w,s_r^a,0)}^{\tilde{w}} \left(V\left(x,s_r^a,1\right) - V\left(w,s_r^a,0\right) - c_j\right) f\left(x\right) dx
\end{aligned}
\tag{3.12}
$$

Similar to the other cases, the optimal reservation place utility inducing a residence move to the region abroad depends on the expectation of future job and residence offers. This reservation place utility, however, is not unambiguously higher than the current place utility because, first of all, a move to the region of work decreases the commuting distance. Hence a residence move not only serves to improve the place utility but also to decrease commuting costs. Second, the move of residence implies changes in the expected lifetime utilities when accepting a new job at home or abroad. However, the changes in lifetime utilities when accepting a new job at home or abroad can be shown to imply an overall positive compensation term, leaving the decrease in the commuting distance as the only negative compensation term.

The terms in (3.12) are interpreted as follows:

1) The term $\left(c_r^h + c_r^a\right)\delta$ is the long run compensation of incurring the residence moving cost, $c_r^h + c_r^a$.

2) The worker takes into account that after accepting a residence offer in the region abroad, he receives other residence offers in the (new) home region, at the rate p_r. Rewriting the terms of future residence offers, we get:

$$p_r \int_{s_r^a(w,r,1)}^{\tilde{r}} \left(V(w,x,0) - V(w,s_r^a,0)\right) g(x)\, dx -$$

$$p_r \int_{s_r^h(w,s_r^a,0)}^{\tilde{r}} \left(V(w,x,0) - V(w,s_r^a,0) - c_r^h\right) g(x)\, dx$$

$$=$$

$$c_r^h p_r \left(1 - G\left(s_r^h(w,s_r^a,0)\right)\right) +$$

$$p_r \int_{s_r^a(w,r,1)}^{s_r^h(w,s_r^a,0)} \left(V(w,x,0) - V(w,s_r^a,0)\right) g(x)\, dx$$

The term $c_r^h p_r \left(1 - G\left(s_r^h(w,s_r^a,0)\right)\right)$ can be interpreted as the compensation for leaving the new residence again. And the term $p_r \int_{s_r^a(w,r,1)}^{s_r^h(w,s_r^a,0)} \left(V(w,x,0) - V(w,s_r^a,0)\right) g(x)\, dx$ may be interpreted as "the guard against the possibility of getting another residence offer after changing residence that would have been preferred before changing, but which is not sufficiently high to induce a second change".

3) Finally, the worker takes future job offers into account, and thereby the expectation of future gains due to changing job. If the position at the job market, home or abroad, worsens, in the sense that the expected future gain due to changing job is lower after having accepted a new residence in the home region compared to the current job market situation, the compensation is positive, otherwise it is negative.

It is shown in Appendix A that, the minimum acceptable wage inducing a job change in the new home region after having changed residence is greater than the minimum acceptable wage inducing a job change in the former home region, i.e. $s_j^h(w,s_r^a,0) > s_j^h(w,r,1)$. This implies that the terms of future job offers in the home region can be rewritten:

$$
\begin{aligned}
&p_j^h \int_{s_j^h(w,r,1)}^{\tilde{w}} \left(V\left(x,r,0\right) - V\left(w,r,1\right) - c_j\right) f\left(x\right) dx - \\
&p_j^h \int_{s_j^h(w,s_r^a,0)}^{\tilde{w}} \left(V\left(x,s_r^a,0\right) - V\left(w,s_r^a,0\right) - c_j\right) f\left(x\right) dx + \\
&= \\
&p_j^h \int_{s_j^h(w,r,1)}^{s_j^h(w,s_r^a,0)} \left(V\left(x,r,0\right) - V\left(w,r,1\right) - c_j\right) f\left(x\right) dx + \\
&p_j^h \int_{s_j^h(w,s_r^a,0)}^{\tilde{w}} \left(V\left(x,r,0\right) - V\left(x,s_r^a,0\right) + c_r^h + c_r^a\right) f\left(x\right) dx
\end{aligned}
\tag{3.13}
$$

The expected future job market situation in the home region (the new respectively the old home region) thus implies a positive compensation term, to guard against the possibility of getting a wage offer after changing residence that would have been preferred before changing, but which is not sufficiently high to induce a job change after changing residence. And, in addition, there is a compensation term due to the change in the expected lifetime utility when accepting a new job in the home region, which would also have been accepted if no change of residence had occurred. This compensation can be shown to be positive, no matter whether s_r^a is greater than r or vice versa, see Appendix A. Consequently, taking the expected future position at the job market in the home region into account implies an overall positive compensation.

Likewise, it is shown in Appendix A that, the minimum acceptable wage inducing a job change in the new region abroad after having changed residence is greater than the minimum acceptable wage inducing a job change in the former region abroad, i.e. $s_j^a\left(w,s_r^a,0\right) > s_j^a\left(w,r,1\right)$. This implies that the terms of future job offers in the regions abroad can be rewritten:

$$
\begin{aligned}
& p_j^a \int_{s_j^a(w,r,1)}^{\tilde{w}} \left(V\left(x,r,1\right) - V\left(w,r,1\right) - c_j\right) f\left(x\right) dx - \\
& p_j^a \int_{s_j^a(w,s_r^a,0)}^{\tilde{w}} \left(V\left(x,s_r^a,1\right) - V\left(w,s_r^a,0\right) - c_j\right) f\left(x\right) dx \\
& = \\
& p_j^a \int_{s_j^a(w,r,1)}^{s_j^a(w,s_r^a,0)} \left(V\left(x,r,1\right) - V\left(w,r,1\right) - c_j\right) f\left(x\right) dx + \\
& p_j^a \int_{s_j^a(w,s_r^a,0)}^{\tilde{w}} \left(V\left(x,r,1\right) - V\left(x,s_r^a,1\right) + c_r^h + c_r^a\right) f\left(x\right) dx
\end{aligned}
\tag{3.14}
$$

Taking account of the expected future job market situation in the region abroad (the new respectively the old region abroad) implies a positive compensation term, to guard against the possibility of getting a wage offer after changing residence that would have been preferred before changing, but which is not sufficiently high to induce a job change after changing residence. And, in addition, there is a compensation term due to the change in the expected lifetime utility when accepting a new job in the region abroad, which would also have been accepted if no change of residence had occurred. This compensation can be shown to be positive no matter whether s_r^a is greater than r or vice versa, see Appendix A. The overall compensation due to job search abroad is thus also positive.

Consequently, the solution to the minimum acceptable residence offer in the region abroad may be rewritten:

$$
\begin{aligned}
u\left(w,s_r^a\right) &= u\left(w,r\right)-c\left(1\right)+\left(c_r^h+c_r^a\right)\delta+ \\
&c_r^h p_r\left(1-G\left(s_r^h\left(w,s_r^a,0\right)\right)\right)+ \\
&p_r\int_{s_r^a(w,r,1)}^{s_r^h(w,s_r^a,0)}\left(V\left(w,x,0\right)-V\left(w,s_r^a,0\right)\right)g\left(x\right)dx+ \\
&p_j^h\int_{s_j^h(w,r,1)}^{s_j^h(w,s_r^a,0)}\left(V\left(x,r,0\right)-V\left(w,r,1\right)-c_j\right)f\left(x\right)dx \\
&p_j^h\int_{s_j^h(w,s_r^a,0)}^{\tilde{w}}\left(V\left(x,r,0\right)-V\left(x,s_r^a,0\right)+c_r^h+c_r^a\right)f\left(x\right)dx+ \\
&p_j^a\int_{s_j^a(w,r,1)}^{s_j^a(w,s_r^a,0)}\left(V\left(x,r,1\right)-V\left(w,r,1\right)-c_j\right)f\left(x\right)dx+ \\
&p_j^a\int_{s_j^a(w,s_r^a,0)}^{\tilde{w}}\left(V\left(x,r,1\right)-V\left(x,s_r^a,1\right)+c_r^a\right)f\left(x\right)dx
\end{aligned}
\tag{3.15}
$$

The compensation given by (3.15) may be positive or negative, depending on the magnitude of the commuting cost.

To sum up, the worker's behaviour is characterised by the 4 reservation values given in the equations: (3.4), (3.6), (3.10), and (3.15). Of these, the value of the offers inducing a job change, both in the home region and abroad, as well as a change of residence in the home region, are unambiguously higher than the current values. Only the minimum acceptable residence offer in the region abroad may be lower than the current place utility, as this change also serves to reduce the commuting cost.

4. Labour market policy

One thing is the model describing the search behaviour of individuals taking both the decision to change job and to change residence into account, simultaneously. Another thing, however, is the scope left for labour market policies. From the policy makers point of view, information on for instance the effect of job and residence moving costs is relevant. In this section we discuss what the analysis of the comparative statics of the model can contribute with.

It is clear from the analysis above, that the interaction between job search and residence search is very complex. On the other hand, a complex setup is also realistic in the sense that people seldom make partial important decisions.

The implementation of the optimal strategies will be discussed based on "base-line" cases. By varying the job and residence offer arrival rates, the costs of moving and the initial job and residence utility, we are able to induce for example how higher educated workers are likely to behave in this setup compared to less educated workers, or how unemployed workers act compared to employed workers. The results of the comparative statics analysis are summarised in the tables below:

Table 4.1 Comparative static results

	w	r
s_j^h	$+$	0
s_j^a	$+$	0
s_r^h	0	$+$
s_r^a	0	$+$
$s_j^h - w$	$+$	0
$s_j^a - w$	$+$	0
$s_r^h - r$	0	$+$
$s_r^a - r$	0	?

Table 4.2 Comparative static results

	c_j	$c_r(0)$	$c_r(1)$	p_j^h	p_j^a	p_r
s_j^h	$+$	?	?	$+$	$+$	?
s_j^a	$+$	$-$	$+$	?	$+$	$-$
s_r^h	?	$+$	?	?	?	$+$
s_r^a	?	$+$	$+$	?	?	?

The first set of results show, that mobility is dependent on where the worker is placed in the distribution of wages or place utilities. Thus, for instance it can be shown that the reservation wages, both in the home region and abroad, not only are increasing in the current wage, but furthermore, that the differences increase with the level of the current wage[4]:

$$\frac{d\left(s_j^h(w,r,0)-w\right)}{dw}>0$$

$$\frac{d\left(s_j^a(w,r,0)-w\right)}{dw}>0$$

Thus, getting closer and closer to the upper bound of the wage distribution implies that the job offer has to be increasingly favorable to induce a job change.

A similar result is found for the home region reservation place utility - getting closer and closer to the upper bound of the place utility distribution implies that the residence offer has to be increasingly favorable to induce a residence move:

$$\frac{d\left(s_r^h(w,r,0)-r\right)}{dr}>0$$

Especially the effects of the job and residence moving costs are interesting to analyse. Using the technique for qualitative comparative statics analysis as proposed in Albrecht et al. (1991)[5] we are able to show that the minimum acceptable wage and place utility inducing a move in the home region is increasing in the job respectively the residence moving cost, i.e. $\frac{ds_j^h}{dc_j}>0$ and $\frac{ds_r^h}{dc(0)}>0$. Hence, the higher the once and only cost of changing job and residence in the home region the more favorable a given offer has to be to be acceptable. Consequently less job and residence mobility in the home region will occur.

Furthermore, we are able to show that the minimum acceptable wage offer inducing a job change to the region abroad is increasing in all of the moving costs, except the cost of moving residence in the home region[6], i.e. $\frac{ds_j^a}{dc_j}>0$, $\frac{ds_j^a}{dc_r(1)}>0$ and $\frac{ds_j^a}{dc_r(0)}<0$. Hence if the cost of changing job, either in the home region or abroad, as well as the cost of moving residence to the region abroad, increase, the more favorable an acceptable job offer from

[4]This result is contrary to the result in Burgess (1992) and is based on the assumption that the instantaneous utility function $u(w,r)$ is strictly concave in the wage.

[5]Albrecht et al. (1991) show that it is possible to establish qualitative properties of the value function V with respect to exogenous parameters, by use of mathematical induction.

[6]The proofs of all results are given in the Appendix.

abroad has to be. Consequently the amount of interregional job mobility decreases. It is intuitively straightforward that a higher job moving cost decreases the job mobility rate. A higher residence moving cost implies that after accepting a job offer from abroad, reducing the increased commuting cost becomes more costly. This explains why the job offer from the region abroad has to be more favorable to be acceptable, i.e. that $\frac{ds_j^a}{dc_r(1)}$ is positive. A higher residence moving cost in the home region on the other hand reduces the least acceptable job offer from abroad. The explanation is that it becomes relatively less costly to improve the housing situation when the search area is the region abroad, which by assumption requires accepting a job offer from abroad.[7]

The minimum acceptable place utility inducing a move to the region abroad, the work region, increases with both the residence cost of moving abroad, $\frac{ds_r^a}{dc(1)} > 0$, as well as with the residence cost of moving in the home region, $\frac{ds_r^a}{dc(0)} > 0$. The first result is intuitively straightforward. Concerning the second result the explanation is, that the possibility of accepting a low-value residence offer, $x < r$, in the region abroad in order to reduce the commuting cost and then afterwards start searching for a more suitable residence offer, $x \geqslant r$, in the new home region, becomes more costly. It is thus more important that the first residence offer from the work region is of a high value.

Concerning the job offer arrival rates we are able to show that $\frac{ds_j^h}{dp_j^h} > 0$, $\frac{ds_j^h}{dp_j^a} > 0$ and $\frac{ds_j^a}{dp_j^a} > 0$.

Thus, if the supply of jobs is high, in the home region as well as abroad, the more favorable an acceptable job offer from the home region has to be. The reason is that, as it is costly to change job it pays to be patient and wait for the high job offer to arrive, because time until a new job offer arrives is short and hence the lost income from not accepting a less favorable job offer is low.

Even though a higher reservation wage decreases job mobility, the overall effect on job mobility is ambiguous, as a higher offer arrival rate increases mobility. The least acceptable job offer from the region abroad increases with the job offer arrival rate from

[7] Bear in mind that after a job change to the region abroad the position in the housing market improves in the sense that that the expected future gain due to moving residence is higher, implying the compensation term regarding the housing situation is negative, see equation (3.7).

the region abroad for the same reasons as given above. However we can not determine the effect of an increase in the job offer arrival rate from the home region, the region of the worker's residence, $\frac{ds_j^a}{dp_j^h} \gtrless 0$.

Finally, we are able to show that the minimum acceptable residence offer inducing a move in the home region increases with the residence offer rate: $\frac{ds_r^h}{dp_r} > 0$. However, the minimum acceptable wage offer inducing a job change to the region abroad decreases with the residence offer rate: $\frac{ds_j^a}{dp_r} < 0$. The reason is that it is easier to find an acceptable residence in the new work region and hence reduce the commuting cost. Thus a higher residence offer arrival rate, unambiguously increases job mobility.

5. Conclusion

We have analyzed the job and residence changing behaviour of workers in a model where search takes place in the labour market and housing market simultaneously. We assume it is costly to change job and/or residence. The labour market is assumed characterized by non-overlapping regional commuting areas. Consequently the commuting distance is only affected if a new job is accepted in the region abroad. The commuting distance can only be reduced by moving residence to the region of work or by receiving an acceptable new job offer in region of residence. However, as changing either job or residence is costly, the expected future labour market as well as housing market situation has to be taken into account before deciding to accept a job offer abroad.

In general, the higher the current wage, the more favorable an acceptable job offer, home as well as abroad, has to be. Likewise, the higher the current place utility is, the more favorable an acceptable residence offer, in the home region as well as in the work region abroad, has to be.

When deciding to change job (residence) the worker takes into account the change in the expected future gains due to changing residence (job). If the position in the housing (job) market worsens, in the sense that the expected future gain due to changing residence (job) is lower after having accepted a new job (residence) compared to the current housing (job) market situation, the compensation is positive, otherwise it is negative. However, the expected future housing market situation in the home region is not affected when the minimum acceptable job offer in the home region is considered. Likewise, the expected future job market situation in the home region is not affected when the minimum acceptable residence offer in the home region is considered. Only positive compensation terms due to future (residence) job offers are present in the determination of the minimum acceptable (residence) job offer in the home region.

We are able to show that when the once and only costs of moving job and/or residence are high, then in general, the rate of job as well as residence mobility will be low. Especially we are able to show that the probability of accepting a job offer in the region abroad decreases with the job moving cost as well as the cost of moving residence to the region abroad.

Finally we show that a high supply of jobs from regions abroad, increases the reservation wage. The effect on job mobility is ambiguous though, as a higher reservation wage decreases job mobility, whereas a higher offer arrival rate increases mobility.

A. Appendix

By assumption the instantaneously utility function $u(w, r)$ is separable in the wage and place utility:

$$u_{wr} = 0 \tag{A.1}$$

The derivatives of the lifetime utility with respect to the wage and place utility are: (see Stokey, Lucas and Prescott 1993.)

$$\delta V_w = u_w, \;\; \delta V_r = u_r \tag{A.2}$$

Using (A.1) and (A.2) it follows from differentiation of the first order conditions, equations (3.2), (3.5), (3.8) and (3.11), that the reservation wages respectively place utilities are independent of the initial place utility respectively wage:

$$\frac{ds_j^h}{dr} = \frac{V_r(w, r, 0) - V_r\left(s_j^h, r, 0\right)}{V_{s_j^h}\left(s_j^h, r, 0\right)} = 0 \tag{A.3}$$

$$\frac{ds_j^a}{dr} = \frac{V_r(w, r, 0) - V_r\left(s_j^a, r, 1\right)}{V_{s_j^a}\left(s_j^a, r, 1\right)} = 0 \tag{A.4}$$

$$\frac{ds_r^h}{dw} = \frac{V_w(w, r, 0) - V_w\left(w, s_r^h, 0\right)}{V_{s_r^h}\left(w, s_r^h, 0\right)} = 0 \tag{A.5}$$

$$\frac{ds_r^a}{dw} = \frac{V_w(w, r, 1) - V_w\left(w, s_r^a, 0\right)}{V_{s_r^a}\left(w, s_r^a, 0\right)} = 0 \tag{A.6}$$

In order to show that: $s_j^h(w, r, 1) < s_j^h(w, s_r^a, 0)$, $s_j^a(w, r, 1) < s_j^a(w, s_r^a, 0)$ and $s_r^a\left(s_j^a, r, 1\right) < s_r^h(w, r, 0)$, we make use of the general result: consider two strictly concave functions, $F(x, \;)$ and $G(x, \;)$. If $F_x < G_x$ for all x then $x_1^* < x_2^*$, where x_1^* and x_2^* are the interior solutions to $\max_x F(x, \;)$ and $\max_x G(x, \;)$.

Using the first order conditions of s_j^h, equation (3.2), we have:

$$
\begin{aligned}
-V(x,r,0)+V(w,r,1)+c_j+V(x,s_r^a,0)-V(w,s_r^a,0)-c_j &= \\
-c_r^a-c_r^h+V(x,s_r^a,0)-V(x,r,0) &= \\
V(x,r,1)-V(x,r,0) &< 0 \Rightarrow \\
s_j^h(w,r,1) &< s_j^h(w,s_r^a,0)
\end{aligned} \tag{A.7}
$$

where the first and second equality uses the first order condition of s_r^a, equation (3.11). In general $V(w,r,1) < V(w,r,0)\ \forall w,r$, which can be proven by contradiction.

Using the first order condition of s_j^a, equation (3.5) we have the result:

$$
\begin{aligned}
-V(x,r,1)+V(w,r,1)+c_j+V(x,s_r^a,1)-V(w,s_r^a,0)-c_j &= \\
-c_r^a-c_r^h-V(x,r,1)+V(x,s_r^a,1) &= \\
V(x,s_r^a,1)-V(x,s_r^a,0) &< 0 \Rightarrow \\
s_j^a(w,r,1) &< s_j^a(w,s_r^a,0)
\end{aligned} \tag{A.8}
$$

where the first and second equality uses the first order condition of s_r^a, equation (3.11).

Using the first order condition of s_r^a, equation (3.11) we have the result:

$$
\begin{aligned}
-V\left(s_j^a,x,0\right)+V\left(s_j^a,r,1\right)+c_r^h+c_r^a+V(w,x,0)-V(w,r,0)-c_r^h &= \\
c_j+c_r^a+V(w,x,0)-V\left(s_j^a,x,0\right) &= \\
c_r^a+V\left(s_j^a,x,1\right)-V\left(s_j^a,x,0\right) &< 0 \Rightarrow \\
s_r^a\left(s_j^a,r,1\right) &< s_r^h(w,r,0)
\end{aligned} \tag{A.9}
$$

where the first and second equality uses the first order condition of s_j^a, equation (3.5).

Finally, using the first order condition of s_j^a and s_j^h, equation (3.2) and (3.5), we have the result:

$$
\begin{aligned}
-V(x,r,0)+V(w,r,0)+c_j+V(x,r,1)-V(w,r,0)-c_j &= \\
-V(x,r,0)+V(x,r,1) &< 0 \Rightarrow \\
s_j^h(w,r,0) &< s_j^a(w,r,0)
\end{aligned} \tag{A.10}
$$

To show that

$$p_r \int_{s_r^h(w,r,0)}^{\tilde{r}} \left(V(w,x,0) - V\left(s_j^a, r, 1\right) + c_j - c_r^h\right) g(x)\, dx - \qquad \text{(A.11)}$$
$$p_r \int_{s_r^a\left(s_j^a,r,1\right)}^{\tilde{r}} \left(V\left(s_j^a, x, 0\right) - c_r^a - c_r^h - V\left(s_j^a, r, 1\right)\right) g(x)\, dx < 0$$

we define $y_0 = V\left(s_j^a, r, 1\right) - c_j + c_r^h$ and $y_1 = V\left(s_j^a, r, 1\right) + c_r^a + c_r^h$ and use the inequality: $s_r^a\left(s_j^a, r, 1\right) < s_r^h(w, r, 0)$ to rewrite (A.11):

$$-p_r \int_{s_r^a\left(s_j^a,r,1\right)}^{s_r^h(w,r,0)} \left(V\left(s_j^a, x, 0\right) - y_1\right) g(x)\, dx + \qquad \text{(A.12)}$$
$$p_r \int_{s_r^h(w,r,0)}^{\tilde{r}} \left(V(w,x,0) - y_0\right) g(x)\, dx -$$
$$p_r \int_{s_r^h(w,r,0)}^{\tilde{r}} \left(V\left(s_j^a, x, 0\right) - y_1\right) g(x)\, dx$$

The first term in (A.12) is negative. The sum of the last two terms is negative, which can be shown graphically. To do that we first notice that:

$$\begin{aligned} -y_0 + y_1 &= \\ c_r^a + c_j &> 0 \end{aligned}$$

where the inequality holds as, by assumption $c_r^a > c_r^h$. Next, we make use of the assumptions that the lifetime utility functions are concave and separable in the current wage and place utility. Consequently the terms $V(w,x,0) - y_0$ and $V\left(s_j^a, x, 0\right) - y_1$, can be depicted as in Figure $A.1$. As can be seen from the figure, $V(w,x,0) - y_0 < V\left(s_j^a, x, 0\right) - y_1$, hence, the sum of the last two terms in (A.12) is negative:

$$p_r \int_{s_r^h(w,r,0)}^{\tilde{r}} \left(V(w,x,0) - y_0\right) g(x)\, dx - p_r \int_{s_r^h(w,r,0)}^{\tilde{r}} \left(V\left(s_j^a, x, 0\right) - y_1\right) g(x)\, dx < 0$$

To show that:

$$p_j^h \int_{s_j^h(w,r,1)}^{\tilde{w}} \left(V(x,r,0) - V(w,r,1) - c_j\right) f(x)\, dx - \tag{A.13}$$
$$p_j^h \int_{s_j^h(w,s_r^a,0)}^{\tilde{w}} \left(V(x,s_r^a,0) - V(w,s_r^a,0) - c_j\right) f(x)\, dx > 0$$

we define $y_0 = V(w,r,1) + c_j$ and $y_1 = V(w,s_r^a,0) + c_j$ and use the inequality: $s_j^h(w,r,1) < s_j^h(w,s_r^a,0)$ to rewrite (A.13):

$$p_j^h \int_{s_j^h(w,r,1)}^{s_j^h(w,s_r^a,0)} \left(V(x,r,0) - y_0\right) f(x)\, dx + \tag{A.14}$$
$$p_j^h \int_{s_j^h(w,s_r^a,0)}^{\tilde{w}} \left(V(x,r,0) - y_0 - \left(V(x,s_r^a,0) - y_1\right)\right) f(x)\, dx$$

The first term in (A.14) is positive. The sum of the last two terms is positive, which can be shown graphically. To do that we first notice that:

$$\begin{aligned} -y_0 + y_1 &= \\ c_r^a + c_r^h &> 0 \end{aligned}$$

Next, we make use of the assumptions that the lifetime utility functions are concave and separable in the current wage and place utility. Consequently the terms $V(x,r,0) - y_0$ and $V(x,s_r^a,0) - y_1$, can be depicted, dependent on whether $s_r^a < r$ or $s_r^a > r$, either as in Figure $A.2$ or Figure $A.3$.

As can be seen from the figures, $V(x,r,0) - y_0 > V(x,s_r^a,0) - y_1$, no matter whether $s_r^a < r$ or $s_r^a > r$. Thus the sum of the last two terms in (A.14) is positive. The same method can be applied to show that:

$$p_j^a \int_{s_j^a(w,r,1)}^{\tilde{w}} \left(V(x,r,1) - V(w,r,1) - c_j\right) f(x)\, dx - \tag{A.15}$$
$$p_j^a \int_{s_j^a(w,s_r^a,0)}^{\tilde{w}} \left(V(x,s_r^a,1) - V(w,s_r^a,0) - c_j\right) f(x)\, dx > 0$$

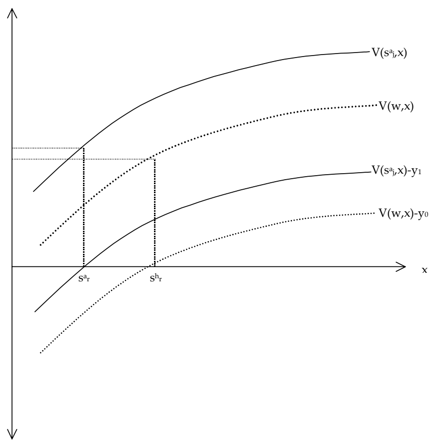

Figure $A.1$

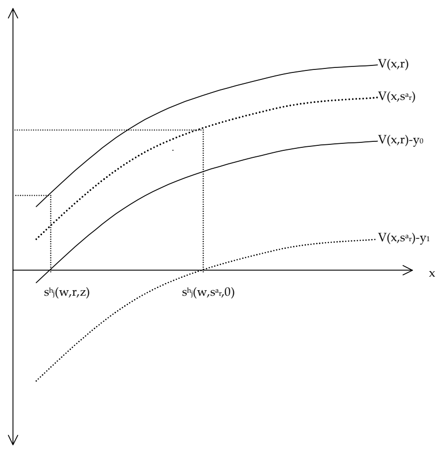

Figure $A.2$

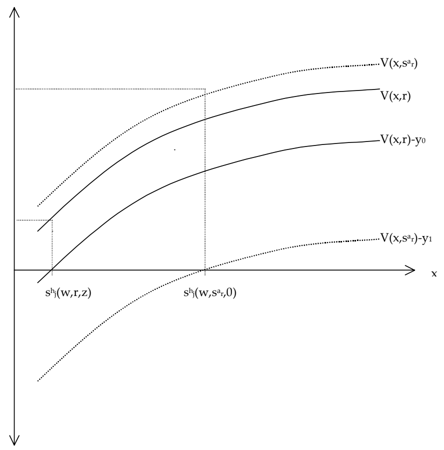

Figure $A.3$

B. Appendix, $\frac{ds_j^h}{dc_j} > 0$

The sign of $\frac{ds_j^h}{dc_j}$ is equal to the sign of $V_{c_j}(w,r,0) - V_{c_j}\left(s_j^h, r, 0\right) + 1$. It is convenient to rewrite the equation giving the value function, equation (3.1) slightly. Choose any constant $m > 0$ such that $m > p_j^h + p_j^a + p_r$ and $m > p_j^h + p_j^a + p_r$. Adding $mV(w,r,0)$ to both sides and dividing through by $m + \delta$ gives:

$$V(w,r,0)=\frac{1}{m+\delta}\left[\begin{array}{c} u(w,r)+p_j^h E_{\max}\left[V(w,r,0),V(x,r,0)-c_j\right]+ \\ p_j^a E_{\max}\left[V(w,r,0),V(x,r,1)-c_j\right]+ \\ p_r E_{\max}\left[V(w,r,0),V(w,x,0)-c_r^h\right]+ \\ \left(m-p_j^h-p_r-p_j^a\right)V(w,r,0) \end{array}\right] \tag{B.1}$$

To show that $\frac{ds_j^h}{dc_j}$ is positive, we first establish a preparatory result:

$$\delta V_{c_j}(w,r,0)<-p_j^h\left(1-F\left(s_j^h\right)\right)+p_j^a\int_{s_j^a}^{\tilde{w}}\left(V_{c_j}(x,r,1)-V_{c_j}(w,r,0)-1\right)f(x)\,dx \tag{B.2}$$

Define:

$$\Omega=-p_j^h\left(1-F\left(s_j^h\right)\right)+p_j^a\int_{s_j^a}^{\tilde{w}}\left(V_{c_j}(x,r,1)-V_{c_j}^n(w,r,0)-1\right)f(x)\,dx \tag{B.3}$$

Hence we need to show that $\delta V_{c_j}^n(w,r,0)<\Omega$ implies $\delta V_{c_j}^{n+1}(w,r,0)<\Omega$, where, using equation (B.1):

$$V_{c_j}^{n+1}(w,r,0)=\frac{1}{m+\delta}\left[\begin{array}{c} p_j^h E V_{c_j}^n\left(\max\left[x\geq s_j^h,w\right],r,0\right)+ \\ p_r E V_{c_j}^n\left(w,\max\left[x\geq s_r^h,r,0\right]\right)+ \\ \Omega+\left(m-p_j^h-p_r\right)V_{c_j}^n(w,r,0) \end{array}\right] \tag{B.4}$$

Multiplying through by δ and using the inductive hypothesis,

$$\delta V_{c_j}^{n+1}(w,r,0)<\frac{1}{m+\delta}\left[\begin{array}{c} \delta\Omega+\left(p_j^h+p_r\right)\Omega \\ +\left(m-p_j^h-p_r\right)\Omega \end{array}\right] \tag{B.5}$$

Cancelling common terms gives:

$$\delta V_{c_j}^{n+1}(w,r,0)<\Omega \tag{B.6}$$

which was to be shown. Next we need to show that $V_{c_j}^n(w,r,0)>V_{c_j}\left(s_j^h,r,0\right)-1$ implies $V_{c_j}^{n+1}(w,r,0)>V_{c_j}\left(s_j^h,r,0\right)-1$. $V_{c_j}^{n+1}(w,r,0)$ is given in equation (B.4). Using the inductive hypothesis gives:

$$V_{c_j}^{n+1}(w,r,0) > \frac{1}{m+\delta}\left[\begin{array}{c} p_j^h\left(V_{c_j}\left(s_j^h,r,0\right)-1\right)+p_r\left(V_{c_j}\left(s_j^h,r,0\right)-1\right) \\ +\Omega+\left(m-p_j^h-p_r\right)\left(V_{c_j}\left(s_j^h,r,0\right)-1\right) \end{array}\right] \tag{B.7}$$

Cancelling common terms and rearranging gives:

$$m\left(V_{c_j}^{n+1}(w,r,0)-V_{c_j}\left(s_j^h,r,0\right)+1\right) > -\delta V_{c_j}^{n+1}(w,r,0)+\Omega > 0 \tag{B.8}$$

where the last inequality follows from equation (B.6).

C. Appendix, $\frac{ds_j^a}{dc_j} > 0$

The sign of $\frac{ds_j^a}{dc_j}$ is equal to the sign of $V_{c_j}(w,r,0) - V_{c_j}\left(s_j^a,r,1\right)+1$, which we want to show is positive. A preparatory result is required:

$$\begin{aligned} \delta V_{c_j}(w,r,0) \quad &\leq \quad p_j^h \int_{s_j^h(w)}^{\tilde{w}} \left(V_{c_j}(x,r,0)-V_{c_j}(w,r,0)-1\right) f(x)\,dx + \\ &\quad\quad p_j^a \int_{s_j^a(w)}^{\tilde{w}} \left(V_{c_j}(x,r,1)-V_{c_j}(w,r,0)-1\right) f(x)\,dx \end{aligned} \tag{C.1}$$

Define:

$$\begin{aligned} \Psi \quad &= \quad p_j^h \int_{s_j^h(w)}^{\tilde{w}} \left(V_{c_j}(x,r,0)-V_{c_j}(w,r,0)-1\right) f(x)\,dx + \\ &\quad\quad p_j^a \int_{s_j^a(w)}^{\tilde{w}} \left(V_{c_j}(x,r,1)-V_{c_j}^n(w,r,0)-1\right) f(x)\,dx \end{aligned} \tag{C.2}$$

To establish that $\delta V_{c_j}(w,r,0) \leq \Psi$ we need to show that $\delta V_{c_j}^n(w,r,0) \leq \Psi$ implies $\delta V_{c_j}^{n+1}(w,r,0) \leq \Psi$, where:

$$V_{c_j}^{n+1}(w,r,0) = \frac{1}{m+\delta}\left[\begin{array}{c} p_r E V_{c_j}^n\left(w,\max\left[x \geq s_r^h,r,0\right]\right) \\ \Psi+\left(m-p_r\right)V_{c_j}^n(w,r,0) \end{array}\right] \tag{C.3}$$

Multiplying through by δ and using the inductive hypothesis,

$$\delta V_{c_j}^{n+1}(w,r,0) \leq \frac{1}{m+\delta}\left[\begin{array}{c} \delta\Psi + p_r\Psi \\ +(m-p_r)\Psi \end{array}\right] \tag{C.4}$$

Cancelling common terms gives:

$$\delta V_{c_j}^{n+1}(w,r,0) \leq \Psi \tag{C.5}$$

which was to be shown. Next we need to show that $V_{c_j}^{n}(w,r,0) > V_{c_j}\left(s_j^a,r,1\right)-1$ implies $V_{c_j}^{n+1}(w,r,0) > V_{c_j}\left(s_j^a,r,1\right)-1$, where $V_{c_j}^{n+1}(w,r,0)$ is given in equation (C.3). Using the inductive hypothesis gives:

$$V_{c_j}^{n+1}(w,r,0) > \frac{1}{m+\delta}\left[\begin{array}{c} \Psi + p_r\left(V_{c_j}\left(s_j^a,r,1\right)-1\right) \\ +(m-p_r)\left(V_{c_j}\left(s_j^a,r,1\right)-1\right) \end{array}\right] \tag{C.6}$$

Cancelling common terms and rearranging gives:

$$m\left(V_{c_j}^{n+1}(w,r,0) - V_{c_j}\left(s_j^a,r,1\right)+1\right) > -\delta V_{c_j}^{n+1}(w,r,0)+\Psi \geq 0 \tag{C.7}$$

where the last inequality follows from equation (C.5).

D. Appendix, $\frac{ds_j^a}{dc_r(1)} > 0$, $\frac{ds_r^a}{dc_r(1)} > 0$

The sign of $\frac{ds_j^a}{dc_r(1)}$ is equal to the sign of: $V_{c_r(1)}(w,r,0) - V_{c_r(1)}\left(s_j^a,r,1\right)$, which we want to show is positive. First we need to show a preparatory result:

$$\delta V_{c_r(1)}(w,r,0) < p_j^a \int_{s_j^a(w)}^{\tilde{w}} \left(V_{c_r(1)}(x,r,1) - V_{c_r(1)}(w,r,0)\right) f(x)\,dx \tag{D.1}$$

Define:

$$\Gamma = p_j^a \int_{s_j^a(w)}^{\tilde{w}} \left(V_{c_r(1)}(x,r,1) - V_{c_r(1)}^{n}(w,r,0)\right) f(x)\,dx \tag{D.2}$$

Hence we need to show that $\delta V_{c_r(1)}^{n}(w,r,0) < \Gamma$ implies $\delta V_{c_r(1)}^{n+1}(w,r,0) < \Gamma$, where:

$$V_{c_r(1)}^{n+1}(w,r,0) = \frac{1}{m+\delta}\left[\begin{array}{c} p_j^h EV_{c_r(1)}^{n}\left(\max\left[x \geq s_j^h, w\right], r, 0\right) + \\ p_r EV_{c_r(1)}^{n}\left(w, \max\left[x \geq s_r^h, r, 0\right]\right) \\ \Gamma + \left(m - p_j^h - p_r\right) V_{c_r(1)}^{n}(w,r,0) \end{array}\right] \tag{D.3}$$

Multiplying through by δ and using the inductive hypothesis,

$$\delta V_{c_r(1)}^{n+1}(w,r,0) < \frac{1}{m+\delta}\left[\begin{array}{c}\delta\Gamma + \left(p_j^h + p_r\right)\Gamma \\ + \left(m - p_j^h - p_r\right)\Gamma\end{array}\right] \tag{D.4}$$

Cancelling common terms gives:

$$\delta V_{c_r(1)}^{n+1}(w,r,0) < \Gamma \tag{D.5}$$

which was to be shown. Next, we need to show that $V_{c_r(1)}^{n}(w,r,0) > V_{c_r(1)}\left(s_j^a, r, 1\right)$ implies $V_{c_r(1)}^{n+1}(w,r,0) > V_{c_r(1)}\left(s_j^a, r, 1\right)$, where:

$$V_{c_r(1)}^{n+1}(w,r,0) = \frac{1}{m+\delta}\left[\begin{array}{c} p_j^h E V_{c_r(1)}^{n}\left(\max\left[x \geq s_j^h, w\right], r, 0\right) + \\ \Gamma + \\ p_r E V_{c_r(1)}^{n}\left(w, \max\left[x \geq s_r^h, r, 0\right]\right) + \\ \left(m - p_j^h - p_r -\right) V_{c_r(1)}^{n}(w,r,0)\end{array}\right] \tag{D.6}$$

Using the inductive hypothesis,

$$V_{c_r(1)}^{n+1}(w,r,0) > \frac{1}{m+\delta}\left[\begin{array}{c}\left(p_j^h + p_r\right) V_{c_r(1)}\left(s_j^a, r, 1\right) + \\ \Gamma + \\ \left(m - p_j^h - p_r\right) V_{c_r(1)}\left(s_j^a, r, 1\right)\end{array}\right] \tag{D.7}$$

Cancelling common terms and rearranging,

$$m\left(V_{c_r(1)}^{n+1}(w,r,0) - V_{c_r(1)}\left(s_j^a, r, 1\right)\right) > \Gamma - \delta V_{c_r(1)}^{n+1}(w,r,0) > 0 \tag{D.8}$$

where the last inequality follows from (D.5).

The sign of $\frac{ds_r^a}{dc_r(1)}$ is equal to the sign of $V_{c_r(1)}(w,r,1) - V_{c_r(1)}(w, s_r^a, 0) + 1$. It is convenient to rewrite the equation giving the value function, equation (3.1) slightly. Choose any constant $m > 0$ such that $m > p_j^h + p_j^a + p_r$ and $m > p_j^h + p_j^a + p_r$. Adding $mV(w,r,1)$ to both sides and dividing through by $m+\delta$ gives:

$$V(w,r,1)=\frac{1}{m+\delta}\left[\begin{array}{c} u(w,r)-c(1)+p_j^h E_{\max}\left[V(w,r,1),V(x,r,0)-c_j\right]+ \\ p_j^a E_{\max}\left[V(w,r,1),V(x,r,1)-c_j\right]+ \\ p_r E_{\max}\left[V(w,r,1),V(w,x,0)-c_r(1)\right]+ \\ \left(m-p_j^h-p_j^a-p_r\right)V(w,r,1) \end{array}\right] \quad \text{(D.9)}$$

We want to show that $V_{c_r(1)}^n(w,r,1)-V_{c_r(1)}(w,s_r^a,0)+1>0$ implies $V_{c_r(1)}^{n+1}(w,r,1)-V_{c_r(1)}(w,s_r^a,0)+1>0$. First we need to show that $\delta V_{c_r(1)}<-p_r(1-G(s_r^a))$, where:

$$\begin{array}{rl} & p_j^h\int_{s_j^h}^{\tilde{w}}\left(V_{c_r(1)}(x,r,0)-V_{c_r(1)}(w,r,1)\right)f(x)\,dx+ \\ \delta V_{c_r(1)}= & p_j^a\int_{s_j^a}^{\tilde{w}}\left(V_{c_r(1)}(x,r,1)-V_{c_r(1)}(w,r,1)\right)f(x)\,dx+ \\ & p_r\int_{s_r^a}^{\tilde{r}}\left(V_{c_r(1)}(w,x,0)-V_{c_r(1)}(w,r,1)\right)g(x)\,dx \end{array} \quad \text{(D.10)}$$

To do that, we need to show that $\delta V_{c_r(1)}^n<-p_r(1-G(s_r^a))$ implies $\delta V_{c_r(1)}^{n+1}<-p_r(1-G(s_r^a))$. $V_{c_r(1)}^n$, derived from equation (D.9), is given by:

$$V_{c_r(1)}^{n+1}=\frac{1}{\delta+m}\left[\begin{array}{c} p_j^h\int_{s_j^h}^{\tilde{w}}\left(V_{c_r(1)}(x,r,0)-V_{c_r(1)}^n(w,r,1)\right)f(x)\,dx+ \\ p_j^a\int_{s_j^a}^{\tilde{w}}\left(V_{c_r(1)}(x,r,1)-V_{c_r(1)}^n(w,r,1)\right)f(x)\,dx+ \\ p_r\int_{s_r^a}^{\tilde{r}}\left(V_{c_r(1)}(w,x,0)-V_{c_r(1)}^n(w,r,1)\right)g(x)\,dx+ \\ mV_{c_r(1)}^n(w,r,1) \end{array}\right] \quad \text{(D.11)}$$

Multiplying by δ, using (D.10) and the inductive hypothesis gives:

$$\begin{array}{rcl} \delta V_{c_r(1)}^{n+1} & < & \frac{1}{\delta+m}\left[-\delta p_r(1-G(s_r^a))-mp_r(1-G(s_r^a))\right] \\ & = & -p_r(1-G(s_r^a)) \end{array} \quad \text{(D.12)}$$

To show that $V_{c_r(1)}^n(w,r,1)-V_{c_r(1)}(w,s_r^a,0)+1>0$ implies $V_{c_r(1)}^{n+1}(w,r,1)-V_{c_r(1)}(w,s_r^a,0)+1>0$, consider again equation (D.11). Using the result in (D.8) gives:

$$V_{c_r(1)}^{n+1}>\frac{1}{\delta+m}\left[\begin{array}{c} p_j^a EV_{c_r(1)}^n\left(\max\left[x\geq s_j^a,w\right],r,1\right)- \\ p_r(1-G(s_r^a)) \\ \left(m-p_j^a\right)V_{c_r(1)}^n(w,r,1) \end{array}\right] \quad \text{(D.13)}$$

Using the inductive hypothesis, cancelling common terms and rearranging gives:

$$m\left(V^{n+1}_{c_r(1)} - V_{c_r(1)} + 1\right) > -p_r\left(1 - G\left(s^a_r\right)\right) - \delta V^{n+1}_{c_r(1)} > 0$$

where the last inequality follows from (D.12).

E. Appendix, $\frac{ds^a_j}{dc_r(0)} < 0$, $\frac{ds^a_r}{dc_r(0)} > 0$, $\frac{ds^h_r}{dc_r(0)} > 0$

The sign of $\frac{ds^a_j}{dc_r(0)}$ is equal to the sign of: $V_{c_r(0)}(w,r,0) - V_{c_r(0)}\left(s^a_j,r,1\right)$. First we recognise that:

$$\begin{aligned}\delta V_{c_r(0)}\left(s^a_j,r,1\right) &= p^h_j \int_{s^h_j\left(s^a_j\right)}^{\tilde{w}} \left(V_{c_r(0)}(x,r,0) - V_{c_r(0)}\left(s^a_j,r,1\right)\right) f(x)\,dx + \\ &\quad p^a_j \int_{s^a_j\left(s^a_j\right)}^{\tilde{w}} \left(V_{c_r(0)}(x,r,1) - V_{c_r(0)}\left(s^a_j,r,1\right)\right) f(x)\,dx + \\ &\quad p_r \int_{s^a_r(r)}^{\tilde{r}} \left(V_{c_r(0)}\left(s^a_j,x,0\right) - V_{c_r(0)}\left(s^a_j,r,1\right)\right) g(x)\,dx \end{aligned} \tag{E.1}$$

Next, we need to show that $V_{c_r(0)}(w,r,0) < V^n_{c_r(0)}\left(s^a_j,r,1\right)$ implies $V_{c_r(0)}(w,r,0) < V^{n+1}_{c_r(0)}\left(s^a_j,r,1\right)$, where:

$$V^{n+1}_{c_r(0)}\left(s^a_j,r,1\right) = \frac{1}{m+\delta}\left[\begin{array}{c} p^h_j \int_{s^h_j\left(s^a_j\right)}^{\tilde{w}} \left(V_{c_r(0)}(x,r,0) - V^n_{c_r(0)}\left(s^a_j,r,1\right)\right) f(x)\,dx + \\ p^a_j \int_{s^a_j\left(s^a_j\right)}^{\tilde{w}} \left(V_{c_r(0)}(x,r,1) - V^n_{c_r(0)}\left(s^a_j,r,1\right)\right) f(x)\,dx + \\ p_r \int_{s^a_r(r)}^{\tilde{r}} \left(V_{c_r(0)}\left(s^a_j,x,0\right) - V^n_{c_r(0)}\left(s^a_j,r,1\right)\right) g(x)\,dx + \\ mV^n_{c_r(0)}\left(s^a_j,r,1\right) \end{array}\right] \tag{E.2}$$

Using the inductive hypothesis,

$$V^{n+1}_{c_r(0)}\left(s^a_j,r,1\right) > \frac{1}{m+\delta}\left[\begin{array}{c} p^h_j \int_{s^h_j\left(s^a_j\right)}^{\tilde{w}} \left(V_{c_r(0)}(x,r,0) - V^n_{c_r(0)}\left(s^a_j,r,1\right)\right) f(x)\,dx + \\ p^a_j \int_{s^a_j\left(s^a_j\right)}^{\tilde{w}} \left(V_{c_r(0)}(x,r,1) - V^n_{c_r(0)}\left(s^a_j,r,1\right)\right) f(x)\,dx + \\ p_r \int_{s^a_r(r)}^{\tilde{r}} \left(V_{c_r(0)}\left(s^a_j,x\right) - V^n_{c_r(0)}\left(s^a_j,r,1\right)\right) g(x)\,dx + \\ mV_{c_r(0)}(w,r,0) \end{array}\right] \tag{E.3}$$

Rearranging gives:

$$
\begin{aligned}
m\left(V^{n+1}_{c_r(0)}\left(s^a_j,r,1\right)-V_{c_r(0)}\left(w,r,0\right)\right) > p^h_j\int_{s^h_j\left(s^a_j\right)}^{\tilde{w}}&\left(V_{c_r(0)}\left(x,r,0\right)-V^n_{c_r(0)}\left(s^a_j,r,1\right)\right)f\left(x\right)dx+\\
p^a_j\int_{s^a_j\left(s^a_j\right)}^{\tilde{w}}&\left(V_{c_r(0)}\left(x,r,1\right)-V^n_{c_r(0)}\left(s^a_j,r,1\right)\right)f\left(x\right)dx+ \qquad \text{(E.4)}\\
p_r\int_{s^a_r(r)}^{\tilde{r}}&\left(V_{c_r(0)}\left(s^a_j,x,0\right)-V^n_{c_r(0)}\left(s^a_j,r,1\right)\right)g\left(x\right)dx-\\
&\delta V^{n+1}_{c_r(0)}\left(s^a_j,r,1\right)\\
&=0
\end{aligned}
$$

where the last equality sign follows from (E.1).From (E.4) it follows that:

$$
\frac{ds^a_r}{dc_r\left(0\right)}=\frac{V_{c_r(0)}\left(s^a_j,r,1\right)-V_{c_r(0)}\left(s^a_j,s^a_r,0\right)}{V_{s^a_r}\left(s^a_j,s^a_r\right)}>0
$$

The sign of $\frac{ds^h_r}{dc_r(0)}$ is equal to the sign of $V_{c_r(0)}\left(w,r,0\right)-V_{c_r(0)}\left(w,s^h_r,0\right)+1$. We want to show that $V^n_{c_r(0)}\left(w,r,0\right)-V_{c_r(0)}\left(w,s^h_r,0\right)+1>0$ implies $V^{n+1}_{c_r(0)}\left(w,r,0\right)-V_{c_r(0)}\left(w,s^h_r,0\right)+1>0$. First we need to show that $\delta V_{c_r(0)}\left(w,r,0\right)<-p_r\left(1-G\left(s^h_r\right)\right)$, where:

$$
\delta V_{c_r(0)}\left(w,r,0\right)=\begin{array}{c}
p^h_j\int_{s^h_j}^{\tilde{w}}\left(V_{c_r(0)}\left(x,r,0\right)-V_{c_r(0)}\left(w,r,0\right)\right)f\left(x\right)dx+\\
p^a_j\int_{s^a_j}^{\tilde{w}}\left(V_{c_r(0)}\left(x,r,1\right)-V_{c_r(0)}\left(w,r,0\right)\right)f\left(x\right)dx+\\
p_r\int_{s^h_r}^{\tilde{r}}\left(V_{c_r(0)}\left(w,x,0\right)-V_{c_r(0)}\left(w,r,0\right)\right)g\left(x\right)dx
\end{array} \qquad \text{(E.5)}
$$

To do that, we need to show that $\delta V^n_{c_r(0)}\left(w,r,0\right)<-p_r\left(1-G\left(s^h_r\right)\right)$ implies $\delta V^{n+1}_{c_r(0)}\left(w,r,0\right)<-p_r\left(1-G\left(s^h_r\right)\right)$. $V^n_{c_r(0)}\left(w,r,0\right)$ is given by:

$$
V^{n+1}_{c_r(0)}\left(w,r,0\right)=\frac{1}{\delta+m}\left[\begin{array}{c}
p^h_j\int_{s^h_j}^{\tilde{w}}\left(V_{c_r(0)}\left(x,r,0\right)-V^n_{c_r(0)}\left(w,r,0\right)\right)f\left(x\right)dx+\\
p^a_j\int_{s^a_j}^{\tilde{w}}\left(V_{c_r(0)}\left(x,r,1\right)-V^n_{c_r(0)}\left(w,r,0\right)\right)f\left(x\right)dx+\\
p_r\int_{s^h_r}^{\tilde{r}}\left(V_{c_r(0)}\left(w,x,0\right)-V^n_{c_r(0)}\left(w,r,0\right)\right)g\left(x\right)dx+\\
mV^n_{c_r(0)}\left(w,r,0\right)
\end{array}\right] \qquad \text{(E.6)}
$$

Multiplying by δ, using (E.5) and the inductive hypothesis gives:

$$\begin{aligned}\delta V^{n+1}_{c_r(0)}(w,r,0) &< \frac{1}{\delta+m}\left[-\delta p_r\left(1-G\left(s^h_r\right)\right)-mp_r\left(1-G\left(s^h_r\right)\right)\right] \\ &= -p_r\left(1-G\left(s^h_r\right)\right)\end{aligned} \tag{E.7}$$

To show that $V^n_{c_r(0)}(w,r,0)-V_{c_r(0)}\left(w,s^h_r,0\right)+1>0$ implies $V^{n+1}_{c_r(0)}(w,r,0)-V_{c_r(0)}\left(w,s^h_r,0\right)+1>0$, consider again equation (E.6). Using the result in (E.4) gives:

$$V^{n+1}_{c_r(0)}(w,r,0) > \frac{1}{\delta+m}\left[\begin{array}{c} p^h_j EV^n_{c_r(0)}\left(\max\left[x\geq s^h_j,w\right],r,0\right)+ \\ p_r EV^n_{c_r(0)}\left(w,\max\left[x\geq s^h_r,r\right],0\right)- \\ p_r\left(1-G\left(s^h_r\right)\right) \\ \left(m-p^h_j-p_r\right)V^n_{c_r(0)}(w,r,0)\end{array}\right] \tag{E.8}$$

Using the inductive hypothesis, cancelling common terms and rearranging gives:

$$m\left(V^{n+1}_{c_r(0)}(w,r,0)-V_{c_r(0)}\left(w,s^h_r,0\right)+1\right) > -p_r\left(1-G\left(s^h_r\right)\right)-\delta V^{n+1}_{c_r(0)}(w,r,0)>0 \tag{E.9}$$

where the last inequality follows from (E.7).

F. Appendix, $\frac{ds^h_j}{dp^h_j}>0$, $\frac{ds^h_j}{dp^a_j}>0$

The sign of $\frac{ds^h_j}{dp^h_j}$ is equal to the sign of $V_{p^h_j}(w,r,0)-V_{p^h_j}\left(s^h_j,r,0\right)$. To show that $\frac{ds^h_j}{dp^h_j}$ is positive, we first establish a preparatory result:

$$\begin{aligned}\delta V_{p^h_j}(w,r,0) &< \int_{s^h_j}^{\tilde{w}}\left(V(x,r,0)-V(w,r,0)-c_j\right)f(x)\,dx+ \\ & p^h_j\int_{s^h_j}^{\tilde{w}}\left(V_{p^h_j}(x,r,0)-V_{p^h_j}(w,r,0)\right)f(x)\,dx+ \\ & p^a_j\int_{s^a_j}^{\tilde{w}}\left(V_{p^h_j}(x,r,1)-V_{p^h_j}(w,r,0)\right)f(x)\,dx\end{aligned} \tag{F.1}$$

Define:

$$\begin{aligned}\Omega_h &= \int_{s_j^h}^{\tilde{w}} \left(V(x,r,0) - V(w,r,0) - c_j\right) f(x)\, dx + \\ & p_j^h \int_{s_j^h}^{\tilde{w}} \left(V_{p_j^h}(x,r,0) - V_{p_j^h}(w,r,0)\right) f(x)\, dx + \\ & p_j^a \int_{s_j^a}^{\tilde{w}} \left(V_{p_j^h}(x,r,1) - V_{p_j^h}(w,r,0)\right) f(x)\, dx\end{aligned} \tag{F.2}$$

Hence we need to show that $\delta V_{p_j^h}^n(w,r,0) < \Omega_h$ implies $\delta V_{p_j^h}^{n+1}(w,r,0) < \Omega_h$, where, using equation (B.1):

$$V_{p_j^h}^{n+1}(w,r,0) = \frac{1}{m+\delta}\left[\begin{array}{c} p_r E V_{p_j^h}^n\left(w, \max\left[x \geq s_r^h, r, 0\right]\right) + \\ \Omega_h + (m - p_r) V_{p_j^h}^n(w,r,0)\end{array}\right] \tag{F.3}$$

Multiplying through by δ and using the inductive hypothesis,

$$\delta V_{p_j^h}^{n+1}(w,r,0) < \frac{1}{m+\delta}\left[\begin{array}{c} \delta\Omega_h + p_r\Omega_h \\ +(m-p_r)\Omega_h\end{array}\right] \tag{F.4}$$

Cancelling common terms gives:

$$\delta V_{p_j^h}^{n+1}(w,r,0) < \Omega_h \tag{F.5}$$

which was to be shown. Next we need to show that $V_{p_j^h}^n(w,r,0) > V_{p_j^h}\left(s_j^h, r, 0\right)$ implies $V_{p_j^h}^{n+1}(w,r,0) > V_{p_j^h}\left(s_j^h, r, 0\right)$. $V_{p_j^h}^{n+1}(w,r,0)$ is given in equation (F.3). Using the inductive hypothesis gives:

$$V_{p_j^h}^{n+1}(w,r,0) > \frac{1}{m+\delta}\left[\begin{array}{c} p_r\left(V_{p_j^h}\left(s_j^h, r, 0\right)\right) \\ +\Omega_h + (m-p_r) V_{p_j^h}\left(s_j^h, r, 0\right)\end{array}\right] \tag{F.6}$$

Cancelling common terms and rearranging gives:

$$m\left(V_{p_j^h}^{n+1}(w,r,0) - V_{p_j^h}\left(s_j^h, r, 0\right)\right) > -\delta V_{p_j^h}^{n+1}(w,r,0) + \Omega_h > 0 \tag{F.7}$$

where the last inequality follows from equation (F.5).

Likewise it can be shown that $\frac{ds_j^h}{dp_j^a} > 0$.

G. Appendix, $\frac{ds_j^a}{dp_r} < 0$, $\frac{ds_r^h}{dp_r} > 0$, $\frac{ds_j^a}{dp_j^a} > 0$

The sign of $\frac{ds_j^a}{dp_r}$ is equal to the sign of: $V_{p_r}(w, r, 0) - V_{p_r}(s_j^a, r, 1)$, which we want to show is negative. First we recognise that:

$$\begin{aligned} \delta V_{p_r}\left(s_j^a, r, 1\right) &= p_j^h \int_{s_j^h\left(s_j^a\right)}^{\tilde{w}} \left(V_{p_r}(x, r, 0) - V_{p_r}\left(s_j^a, r, 1\right)\right) f(x)\, dx \\ & p_j^a \int_{s_j^a\left(s_j^a\right)}^{\tilde{w}} \left(V_{p_r}(x, r, 1) - V_{p_r}\left(s_j^a, r, 1\right)\right) f(x)\, dx + \\ & p_r \int_{s_j^h\left(s_j^a\right)}^{\tilde{w}} \left(V_{p_r}\left(s_j^a, x, 0\right) - V_{p_r}\left(s_j^a, r, 1\right)\right) f(x)\, dx + \\ & \int_{s_r^a}^{\tilde{r}} \left(V\left(s_j^a, x, 0\right) - V_{p_r}\left(s_j^a, r, 1\right) - c_r^h - c_r^a\right) g(x)\, dx \end{aligned} \tag{G.1}$$

Next, we need to show that $V_{p_r}(w, r, 0) < V_{p_r}^{n+1}\left(s_j^a, r, 1\right)$ implies $V_{p_r}(w, r, 0) < V_{p_r}^{n+1}\left(s_j^a, r, 1\right)$, where:

$$V_{p_r}^{n+1}\left(s_j^a, r, 1\right) = \frac{1}{m+\delta} \left[\begin{array}{c} p_j^h \int_{s_j^h\left(s_j^a\right)}^{\tilde{w}} \left(V_{p_r}(x, r, 1) - V_{p_r}^n\left(s_j^a, r, 1\right)\right) f(x)\, dx + \\ p_j^a \int_{s_j^a\left(s_j^a\right)}^{\tilde{w}} \left(V_{p_r}(x, r, 1) - V_{p_r}^n\left(s_j^a, r, 1\right)\right) f(x)\, dx + \\ p_r \int_{s_j^h\left(s_j^a\right)}^{\tilde{w}} \left(V_{p_r}\left(s_j^a, x, 0\right) - V_{p_r}^n\left(s_j^a, r, 1\right)\right) f(x)\, dx + \\ \int_{s_r^a}^{\tilde{r}} \left(V\left(s_j^a, x, 0\right) - V_{p_r}\left(s_j^a, r, 1\right) - c_r^h - c_r^a\right) g(x)\, dx + \\ m V_{p_r}^n\left(s_j^a, r, 1\right) \end{array} \right] \tag{G.2}$$

Using the inductive hypothesis,

$$V_{p_r}^{n+1}\left(s_j^a, r, 1\right) > \frac{1}{m+\delta} \left[\begin{array}{c} p_j^h \int_{s_j^h\left(s_j^a\right)}^{\tilde{w}} \left(V_{p_r}(x, r, 1) - V_{p_r}^n\left(s_j^a, r, 1\right)\right) f(x)\, dx + \\ p_j^a \int_{s_j^a\left(s_j^a\right)}^{\tilde{w}} \left(V_{p_r}(x, r, 1) - V_{p_r}^n\left(s_j^a, r, 1\right)\right) f(x)\, dx + \\ p_r \int_{s_j^h\left(s_j^a\right)}^{\tilde{w}} \left(V_{p_r}\left(s_j^a, x, 0\right) - V_{p_r}^n\left(s_j^a, r, 1\right)\right) f(x)\, dx + \\ \int_{s_r^a}^{\tilde{r}} \left(V\left(s_j^a, x, 0\right) - V\left(s_j^a, r, 1\right) - c_r^h - c_r^a\right) g(x)\, dx + \\ m V_{p_r}(w, r, 0) \end{array} \right] \tag{G.3}$$

Using (G.1) and rearranging, we get

$$m\left(V_{p_r}^{n+1}(w,r,0) - V_{p_r}\left(s_j^a, r, 1\right)\right) > 0 \tag{G.4}$$

The sign of $\frac{ds_r^h}{dp_r}$ is equal to the sign of $V_{p_r}(w,r,0) - V_{p_r}\left(s_r^h, r, 0\right)$. To show that $\frac{ds_r^h}{dp_r}$ is positive, we first establish a preparatory result:

$$\begin{aligned} \delta V_{p_r}(w,r,0) &< \int_{s_r^h}^{\tilde{r}} \left(V(w,x,0) - V(w,r,0) - c(0)\right) g(x)\, dx + \\ & p_r \int_{s_r^h}^{\tilde{r}} \left(V_{p_r}(x,r,0) - V_{p_r}(w,r,0)\right) g(x)\, dx + \\ & p_j^a \int_{s_j^a}^{\tilde{w}} \left(V_{p_r}(x,r,1) - V_{p_r}(w,r,0)\right) f(x)\, dx \end{aligned} \tag{G.5}$$

Define:

$$\begin{aligned} \Omega_r &= \int_{s_r^h}^{\tilde{r}} \left(V(w,x,0) - V(w,r,0) - c(0)\right) g(x)\, dx + \\ & p_r \int_{s_r^h}^{\tilde{r}} \left(V_{p_r}(w,x,0) - V_{p_r}(w,r,0)\right) g(x)\, dx + \\ & p_j^a \int_{s_j^a}^{\tilde{w}} \left(V_{p_r}(x,r,1) - V_{p_r}(w,r,0)\right) f(x)\, dx \end{aligned} \tag{G.6}$$

Hence we need to show that $\delta V_{p_r}^n(w,r,0) < \Omega_r$ implies $\delta V_{p_r}^{n+1}(w,r,0) < \Omega_r$, where, using equation (B.1):

$$V_{p_r}^{n+1}(w,r,0) = \frac{1}{m+\delta} \left[\begin{array}{c} p_j^h E V_{p_r}^n \left(\max\left[x \geq s_j^h, w\right], r, 0\right) + \\ \Omega_r + \left(m - p_j^h\right) V_{p_r}^n(w,r,0) \end{array} \right] \tag{G.7}$$

Multiplying through by δ and using the inductive hypothesis,

$$\delta V_{p_r}^{n+1}(w,r,0) < \frac{1}{m+\delta} \left[\begin{array}{c} \delta \Omega_r + p_j^h \Omega_r \\ + \left(m - p_j^h\right) \Omega_r \end{array} \right] \tag{G.8}$$

Cancelling common terms gives:

$$\delta V_{p_r}^{n+1}(w,r,0) < \Omega_r \tag{G.9}$$

which was to be shown. Next we need to show that $V^n_{p_r}(w,r,0) > V_{p_r}\left(w,s^h_r,0\right)$ implies $V^{n+1}_{p_r}(w,r,0) > V_{p_r}\left(w,s^h_r,0\right)$. $V^{n+1}_{p_r}(w,r,0)$ is given in equation (G.7). Using the inductive hypothesis gives:

$$V^{n+1}_{p_r}(w,r,0) > \frac{1}{m+\delta}\begin{bmatrix} p^h_j\left(V_{p_r}\left(w,s^h_r,0\right)\right) \\ +\Omega_r + \left(m-p^h_j\right)V_{p_r}\left(w,s^h_r,0\right)\end{bmatrix} \tag{G.10}$$

Cancelling common terms and rearranging gives:

$$m\left(V^{n+1}_{p_r}(w,r,0) - V_{p_r}\left(w,s^h_r,0\right)\right) > -\delta V^{n+1}_{p_r}(w,r,0) + \Omega_r > 0 \tag{G.11}$$

where the last inequality follows from equation (G.9).

The sign of $\frac{ds^a_j}{dp^a_j}$ is equal to the sign of: $V_{p^a_j}(w,r,0) - V_{p^a_j}\left(s^a_j,r,1\right)$, which we want to show is positive. First we need to show a preparatory result:

$$\begin{aligned}\delta V_{p^a_j}(w,r,0) \quad < \quad & p^a_j\int_{s^a_j(w)}^{\tilde{w}}\left(V_{p^a_j}(x,r,1) - V_{p^a_j}(w,r,0)\right)f(x)\,dx + \\ & \int_{s^a_j}^{w}\left(\tilde{V}(w,x,1) - V(w,r,0) - c_j\right)g(x)\,dx\end{aligned} \tag{G.12}$$

Define:

$$\begin{aligned}\Phi_j \quad = \quad & p^a_j\int_{s^a_j(w)}^{\tilde{w}}\left(V_{p^a_j}(x,r,1) - V_{p^a_j}(w,r,0)\right)f(x)\,dx + \\ & \int_{s^a_j}^{w}\left(\tilde{V}(w,x,1) - V(w,r,0) - c_j\right)g(x)\,dx\end{aligned} \tag{G.13}$$

Hence we need to show that $\delta V^n_{p^a_j}(w,r,0) < \Phi_j$ implies $\delta V^{n+1}_{p^a_j}(w,r,0) < \Phi_j$, where:

$$V^{n+1}_{p^a_j}(w,r,0) = \frac{1}{m+\delta}\begin{bmatrix} p^h_j E V^n_{p^a_j}\left(\max\left[x \geq s^h_j, w\right], r, 0\right) + \\ p_r E V^n_{p^a_j}\left(w, \max\left[x \geq s^h_r, r\right], 0\right) \\ \Phi_j + \left(m - p^h_j - p_r\right)V^n_{p^a_j}(w,r,0)\end{bmatrix} \tag{G.14}$$

Multiplying through by δ and using the inductive hypothesis,

$$\delta V_{p_j^a}^{n+1}(w,r,0) > \frac{1}{m+\delta}\left[\begin{array}{c}\delta\Phi_j + \left(p_j^h + p_r^h\right)\Phi_j \\ + \left(m - p_j^h - p_r\right)\Phi_j\end{array}\right] \tag{G.15}$$

Cancelling common terms gives:

$$\delta V_{p_j^a}^{n+1}(w,r,0) > \Phi_j \tag{G.16}$$

which was to be shown. Next, we need to show that $V_{p_j^a}^{n}(w,r,0) > V_{p_j^a}\left(s_j^a,r,1\right)$ implies $V_{p_j^a}^{n+1}(w,r,0) > V_{p_j^a}\left(s_j^a,r,1\right)$, where:

$$V_{p_j^a}^{n+1}(w,r,0) = \frac{1}{m+\delta}\left[\begin{array}{c} p_j^h E V_{p_j^a}^{n}\left(\max\left[x \geq s_j^h, w\right], r, 0\right) + \\ \Phi_j + \\ p_r E V_{p_j^a}^{n}\left(w, \max\left[x \geq s_r^h, r\right], 0\right) + \\ \left(m - p_j^h - p_r -\right) V_{p_j^a}^{n}(w,r,0) \end{array}\right] \tag{G.17}$$

Using the inductive hypothesis,

$$V_{p_j^a}^{n+1}(w,r,0) > \frac{1}{m+\delta}\left[\begin{array}{c} \left(p_j^h + p_r\right) V_{p_j^a}\left(s_j^a,r,1\right) + \\ \Phi_j + \\ \left(m - p_j^h - p_r\right) V_{p_j^a}\left(s_j^a,r,1\right) \end{array}\right] \tag{G.18}$$

Cancelling common terms and rearranging,

$$m\left(V_{p_j^a}^{n+1}(w,r,0) - V_{p_j^a}\left(s_j^a,r,1\right)\right) > \Phi_j - \delta V_{p_j^a}^{n+1}(w,r,0) > 0 \tag{G.19}$$

where the last inequality follows from (G.16).

References

[1] Albrecht, J.W., B. Holmlund and H. Lang (1991). Comparative statics in dynamic programming models with an application to job search. Journal of Economic Dynamics and Control, 15: 755-769.

[2] Andersen, A.K. (2000) Commuting Areas in Denmark. AKF Forlaget

[3] Burgess, S.M. (1992) A search model with changing job costs: 'euroslerosis' and unemployment. Oxford Economic Papers, 44,75-88.

[4] Hey, J.D. and C.J. McKenna (1979). To move or not to move. Economica, 46, 175-185.

[5] Ministry of the Environment, (2001) Pendlingen i Danmark år 2000 og udviklingen i 1990'erne. Copenhagen 2001.

[6] Stokey, N.L., R.E. Lucas and E.C. Prescott (1993) Recursive methods in economic dynamics. Cambridge Mass. Harvard University Press.

[7] Van Ommeren, J., P. Rietveld and P. Nijkamp (1998). Spatial Moving Behaviour of Two-earner Households. Journal of Regional Science, 38 (1): 23-41.

[8] Van Ommeren, J., P. Rietveld and P. Nijkamp (1999). Job Moving, Residential Moving, and Commuting: A Search Perspective. Journal of Urban Economics 46: 230-253.

[9] Westerlund, Olle (1997). Employment Opportunities, Wages and Interregional Migration in Sweden 1970-1989. Journal of Regional Science 37 (1): 55-73.