Social welfare effects of educational labour market programmes

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Socialforskningsinstituttet
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Abstract

A number of papers (e.g. Besley and Coate (1992, 1995)) have considered the optimality of ALMP-programmes and especially the deterrence effect, i.e. the feature that participation in ALMP-programmes implies a disutility comparable to disutility for ordinary work. The papers consider the relative levels of benefit rates in ALMP-programmes and in 'passive' public income support. In this paper, we focus on ALMP-programmes with a positive outcome, namely education programmes that raise participants' level of productivity. A' priori it appears difficult to say whether a positive outcome is a motive for subsidizing ALMP-programmes relative to passive support, or whether individuals' self-interest reduces the need to support such programmes. Hence we discuss the relative benefit rates in optimal of social policy. The optimal benefit rate in education programmes turns out to be higher or lower than the passive benefit rate depending on the distribution of characteristics, but, under reasonable assumptions, a passive benefit rate equal to zero is never optimal. The latter is a trivial but relevant result, because it is in opposition to the policy in many countries where ALPM-participation is a condition for obtaining social benefits.

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1. Introduction

A number of papers have considered the role for activation programmes from the point of view of an optimizing government (e.g. Besley and Coate (1992, 1995), Thustrup Kreiner and Tranæs (2003), Cuff (2001), Brett (1998)), typically with the purpose of determining optimal benefit levels in an activation programme as well as an alternative programme (‘passive’ unemployment benefit or social assistance) and most often with focus on the deterrence effect of activation programmes, i.e. the feature that participation in ALMP-programmes implies a disutility comparable to disutility for ordinary work. A main purpose of this literature is to give a theoretical foundation for the use of activation policy. In Rasmussen (2004, 2005) the focus is also on whether passive benefit rates could be zero in optimum – i.e. whether social benefit could be conditioned on ALMP-participation. This is a relevant issue, because in many countries, unemployed people loose their social assistance or unemployment benefit if they do not participate in activation programmes. In this paper, we focus on activation programmes with a productive outcome, namely a higher level of productivity (potential wage) for participants (Brett (1998) considers activation programmes with production of goods). After participation in education programmes, people may therefore obtain a higher wage, and to capture this we consider a two-period model. Participation might be in peoples own interest and hence there might be no need to punish non-participants with a low, possibly zero, passive benefit rate. On the other hand, the government wants to increase incentives to participate in education in order to make otherwise unemployed prefer work. A’ priori we cannot say what the relative size of the benefit rates should be in optimum. The paper present first order conditions so that we can discuss this.

We use a principal-agent approach. The wage rate and the increase in the wage rate that results from education are private knowledge. Individuals choose work, education or the passive benefit in order to maximize utility over the two periods. The government pursues a high level of equality by choosing appropriate levels of the passive and the education benefit rate and a tax rate on workers, but the generosity of the social benefits has to be balanced with total production because the social security system affect labour supply. The education programme and the tax on work leads to overinvestment in education from otherwise employed individuals but may encourage otherwise unemployed individuals to supply labour.

In sections 2 and 3 the basic model of the paper is considered. The productivity can only be improved through the government's education programme. In section 4, we change the model by assuming that individuals can improve productivity in private programmes. Participation in such programmes is assumed to be private knowledge. Therefore the benefit rate in public education programmes cannot be lower than the passive benefit rate.

In the base model, it is assumed that there are no credit constraints so that income in all (both) periods is the maximized by individuals. In section 5, we assume that consumption is equal to income in each period.

The costs and gains of education programmes are evaluated relative to costs and gains of passive benefits. A natural alternative would be to consider such labour market education programmes opposed to general education. A second natural alternative would be to consider education programmes targeted towards people who have been unemployed for some period. Both alternatives require a model with more dynamics than the two-period model in this paper.

2. The model

Individuals are assumed to choose between ordinary work, to receive a social assistance benefit without participation in an education programme ('passive' benefit), or to participate in an education programme and receiving welfare benefit (education benefit). Individuals make this choice in order to maximize utility. Participants in activation programmes increase their future productivity and hence future wage. We consider a simple two-period model to describe the dynamics. Over the two periods, the individuals therefore choose between three 'period-states' in period 1 multiplied by three period-states in period 2. As becomes clear below, a number of these nine states are dominated by other states, and others are precluded by assumption.

We assume that individuals experience disutility if they participate in education programmes. Ordinary work also implies disutility. The gain in terms of productivity is heterogeneous and private knowledge. Also, the wage rate is private knowledge and these two variables are the only private knowledge-variables in the model. We limit the number of private-knowledge-variables to two for tractability and we consider the choice of these two variables as interesting, because they reflect an outcome of the education programmes which can be compared with the outcome of ordinary work. Certainly it appears likely that people face different gains from the programme. It would alternative be relevant to consider the disutility from education as a private knowledge parameter (as in the model of deterrence motivated activation programmes in Rasmussen (2004, 2005)).

We denote the passive benefit rate (net of tax) as b, the education benefit as a, the wage rate w and the lump sum tax levied on workers as t so income net of tax is $w-t$. Work implies a disutility of d, and activation implies disutility g (each identical for each individual). Passive benefit implies no disutility.

In period 2, the wage rate for individuals who participated in activation in period 1 is $w+r$, where r is the gain in productivity/wage due to the education programme.

Table 1 sums up.

Table 1. Income in period 1 and 2 by states

State	Period 1	Period 2
Passive benefit	b	b
Activation	$a-g$	$a-g$
Work	$w-t-d$	$w-t-d$ if no activation in period 1 $w+r-t-d$ if activation in period 1

An individual's utility is the (undiscounted) sum of period-utilities. Each individual choose (in principle) between the nine states that combine passive benefit, education and work in the two periods. We denote the action from an individual by XY where X is the period-state chosen in period 1 and Y is the period-state chosen in period 2, and where $X,Y \in \{B \text{ (passive benefit)}, A \text{ (education)}, W \text{ (work)}\}$. It is clear that

BW is dominated by WW or BB

WB is dominated by WW or BB

WA is dominated by AW

We also assume that each individual can participate in education in only one period, and hence the action AA is not possible. Finally, action BA gives the same total income as action AB, so for convenience we preclude the latter.

This leaves states WW, BB, AW and BA to be considered.

The individual's choice

Action WW is chosen if the following conditions are satisfied

(1) $WW \succ BB \Leftrightarrow 2(w-t-d) > 2b$

(2) $WW \succ AW \Leftrightarrow 2(w-t-d) > (a-g)+(w+r-t-d)$

(3) $WW \succ BA \Leftrightarrow 2(w-t-d) > b+(a-g)$

Action BB is chosen if the following conditions are satisfied

(1') $BB \succ WW \Leftrightarrow 2b > 2(w-t-d)$

(4) $BB \succ AW \Leftrightarrow 2b > (a-g)+(w+r-t-d)$

(5) $BB \succ BA \Leftrightarrow 2b > b+a-g$

Action AW is chosen if the following conditions are satisfied

(2') $AW \succ WW \Leftrightarrow (a-g)+(w+r-t-d) > 2(w-t-d)$

(4') $AW \succ BB \Leftrightarrow (a-g)+(w+r-t-d) > 2b$

(6) $AW \succ BA \Leftrightarrow (a-g)+(w+r-t-d) > b+(a-g)$

Action BA is chosen if the following conditions are satisfied

(3’) $BA \succ WW \Leftrightarrow b+(a-g) > 2(w-t-d)$

(5’) $BA \succ BB \Leftrightarrow b+(a-g) > 2b$

(6’) $BA \succ AW \Leftrightarrow b+(a-g) > (a-g)+(w+r-t-d)$

Each individual is characterised by her specific values of w and r. Figures 1 and 2 below illustrate the sets of individuals grouped by their choice of state according to their private-knowledge parameters and the policy parameters.

Throughout the paper we consider the case $b > a-g$ and hence $BB \succ BA$ and the reverse case separately.

Figure 1. Combinations of r and w that separates states (case $b > a - g$)

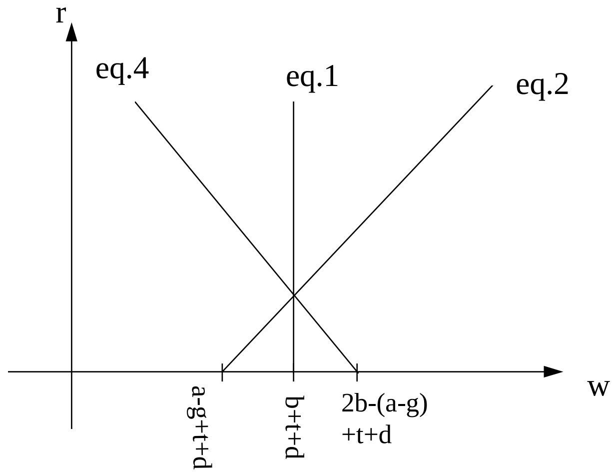
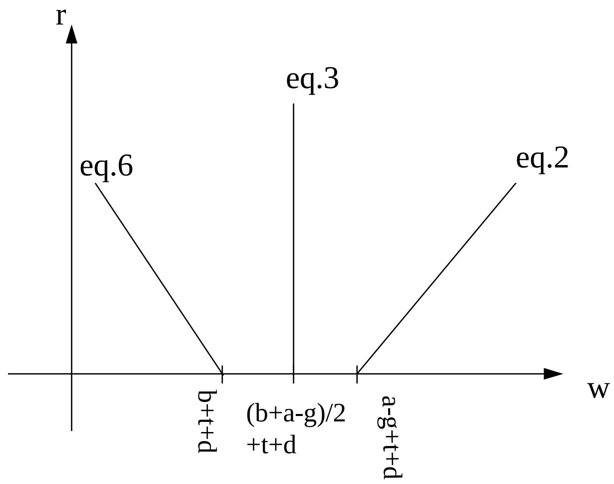


Figure 2. Combinations of r and w that separates states (case $b < a - g$)



In the following we denote by letters in script case, i.e. $\mathcal{W}$, $\mathcal{B}$, and $\mathcal{A}$, the set of individuals who work, obtain passive benefit or participate in education. Let the density and distribution function of these be denoted f and F, and let $W, A,$ and B denote the measures of $\mathcal{W}$, $\mathcal{B}$, and $\mathcal{A}$. We have

$$\begin{aligned}
&\mathcal{W} = \{(w,r):\ \text{action } WW \text{ is preferred}\} \\
&W = \text{measure of } \mathcal{W} = \int_{(w,r)\in\mathcal{W}} f(w,r)\mathrm{d}(w,r) \\
&\mathcal{B} = \begin{cases} \{(w,r):\ \text{action } BB \text{ is preferred}\} & \text{in case } b > a-g \\ \{(w,r):\ \text{action } BA \text{ is preferred}\} & \text{in case } b < a-g \end{cases} \\
&B = \text{measure of } \mathcal{B} = \int_{(w,r)\in\mathcal{B}} f(w,r)\mathrm{d}(w,r) \\
&\mathcal{A} = \{(w,r):\ \text{action } AW \text{ is preferred}\} \\
&A = \text{measure of } \mathcal{A} = \int_{(w,r)\in\mathcal{A}} f(w,r)\mathrm{d}(w,r)
\end{aligned} \tag{7}$$

These sets and the measures of these sets are function of the politically determined variables, $b, a,$ and t.

The benevolent government

The benevolent government maximizes a welfare function which is a function of the utility of each individual. The government weights each individual's income via the concave function u. The curvature of u reflects the government's preference for equality – i.e. ceteris paribus the government prefers completely equal levels of utility, but greater equality reduces the total production because higher levels of benefit rates (b and possibly a) make some individuals prefer not to work.

To write the welfare function, it is useful to denote averages of utilities for individuals in particular states as for example

(8) $$\overline{u^{w}} = \frac{1}{W} \int_{(w,r)\in\mathcal{W}} u(2(w-t-d))\, f(w,r)\mathrm{d}(w,r)$$

Also, partial derivatives of these measures are relevant, for example

(9) $$\overline{u'^{A}} = \frac{1}{A} \int_{(w,r)\in\mathcal{A}} u'(a-g+w+r-t-d)\, f(w,r)\mathrm{d}(w,r)$$

Note that we keep A and $\mathcal{A}$ fixed in (9). Thus $\overline{u'^{A}}$ is the change in utility for individuals who 'initially' participate in education.

The welfare function is therefore

(10) $$\begin{aligned} V &= u(2b)B + \overline{u^{A}}A + \overline{u^{W}}W \quad \text{in case } b > a-g \\ V &= u(b+a-g)B + \overline{u^{A}}A + \overline{u^{W}}W \quad \text{in case } b < a-g \end{aligned}$$

The budget constraint for the government is

(11) $$\begin{aligned} F &= 2Wt - Aa + At - 2Bb = 0 \quad \text{in case } b > a-g \\ F &= 2Wt - Aa + At - B(b+a-t) = 0 \quad \text{in case } b < a-g \end{aligned}$$

The government's problem is to maximize V wrt. b, a and t, and subject to (11). In the next section, we will consider partial derivatives derived from the government's problem. The purpose is to compare the optimal level of b relative to a rather than the levels of b and a relative to t. This facilitates the analysis, and allows us not to solve for t and a Lagrange parameter, see below.

Individuals' behaviour in response to changes in policy variables are of course central elements in the government's problem. Individuals behaviour are captured by derivatives of the measures of sets with respect to policy variables, for example $\frac{\partial W}{\partial b}$. The size of these derivatives depends on 'primitives' of the model, namely the distribution of characteristics, and in general we have little to say about that distribution. Therefore, in the next section, we analyse optimal policy without any attempts to calculate these derivatives. We rather discuss optimal policy in relation to this behaviour of individuals. In figures 3 and 4, the states and the derivatives of measures of the states are illustrated. The derivatives of the measures are the density of the line segments that separates the states, see Rasmussen (2004). Also note that $\frac{\partial W}{\partial b}+\frac{\partial B}{\partial b}+\frac{\partial A}{\partial b}=0$ (similarly for a) and $\frac{\partial B}{\partial a}=\frac{\partial A}{\partial b}$.

Figure 3. **States and derivatives of measures of states** (case $b > a - g$)

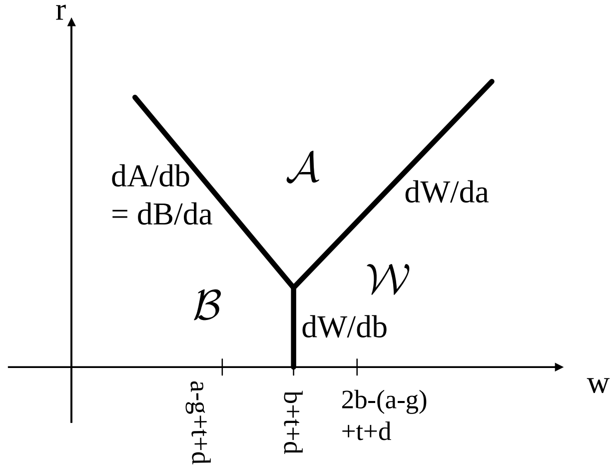
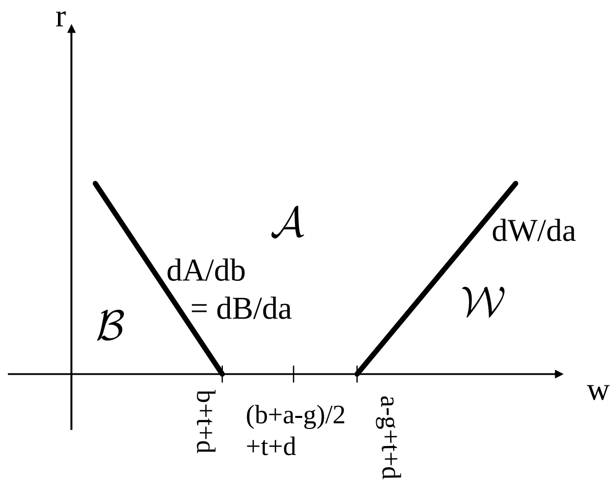


Figure 4. **States and derivatives of measures of states** (case $b < a - g$)



The change in the public budget in response to changes in benefit levels is central in the analysis. The budget is influenced in three ways

- By a direct effect: an increase in (say) b increase expenditures because individuals already obtaining the benefit now obtains a greater benefit.
- A labour supply effect: an increase of b (a) makes some individuals move from $\mathcal{W}$ to $\mathcal{B}$ (to $\mathcal{A}$)
- A programme shifting effect: an increase of b (a) makes some individuals move from $\mathcal{B}$ to $\mathcal{A}$ ($\mathcal{A}$ to $\mathcal{B}$)

We make the following assumptions which are technically useful in the next sections as well as they appear realistic

$$\text{(12)} \qquad \begin{aligned} &\text{Min}\left\{B(a,b,t): a,b,t \text{ fulfills (11)}\right\} > 0 \\ &\text{Min}\left\{A(a,b,t): a,b,t \text{ fulfills (11)}\right\} > 0 \end{aligned}$$

The assumptions say that whatever policy the government chooses, there will always be some individuals who choose passive benefits and some who choose the education programme. This means that even if $b = 0$ some individuals with high levels of work- and education disutility will choose a passive ‘support’ (a misnomer since they will receive zero support), and some individuals will participate in education programmes even if $a = 0$ because the return, r, is high. Technically, this is useful below because it allows us to divide by the measures A and B.

Finally, in the analysis of first order conditions in the next section, we ignore the change of the levels of utility of individuals who move from one state to another (i.e. individuals placed on line segments in figure 3 or 4 that separate states). For example, when we consider a marginal increase of b, we ignore the change in the utility level of a person who transits from work to passive benefit. We do so, because the change of 'movers'' utilities have zero weight because an individual on the border between two states have same utility in neighbouring states and because the borders have zero measure, see Rasmussen (2004).

3. Necessary conditions for optimal policy (b, a and t)

We consider the first order conditions for a solution to the government's problem. We consider the two cases $b > a - g$ and vice verse separately.

Case $b > a - g$ (figures 1 and 3)

First order conditions become (F_z , $z = a, b$, denote the partial derivatives of F)

$$\frac{\partial V}{\partial a} = \overline{u'^A} A - \lambda F_a = 0$$

(13) and

$$\frac{\partial V}{\partial b} = u'(2b) 2B - \lambda F_b = 0$$

The Lagrange variable λ is negative (note that F_b is negative – at least in optimum), and is the shadow value of raising tax revenue by one unit. The way to proceed is to inspect F_a and F_b in more detail. We use $\frac{\partial W}{\partial b}+\frac{\partial A}{\partial b}+\frac{\partial B}{\partial b}=0$ and similarly for a. It is also clear that $\frac{\partial W}{\partial b},\frac{\partial A}{\partial b},\frac{\partial W}{\partial a},\frac{\partial B}{\partial a}\leq 0,$ and $\frac{\partial A}{\partial b},\frac{\partial A}{\partial a}\geq 0$. We have

$$
\begin{aligned}
F_b &= t\frac{\partial W}{\partial b}2-\frac{\partial A}{\partial b}a+\frac{\partial A}{\partial b}t-2b\frac{\partial B}{\partial b}-2B = 2(b+t)\frac{\partial W}{\partial b}+(2b-(a-t))\frac{\partial A}{\partial b}-2B \\
F_a &= t\frac{\partial W}{\partial a}2-\frac{\partial A}{\partial a}a-A+\frac{\partial A}{\partial a}t-2b\frac{\partial B}{\partial a}-A = (a+t)\frac{\partial W}{\partial a}-\left(2b-(a-t)\right)\frac{\partial B}{\partial a}-A
\end{aligned} \qquad (14)
$$

In both equations, the third term is the direct effect. The first term is the labour supply effect. This will increase public net expenditures due to loss of tax payments from workers in both periods ($2t$) and increased benefits ($2b$ or a), but the term increase tax payments from participants in education in period 2 ($-t$).

The second term is the programme shifting effect. A person who enters B increase public expenditures by $2b$ and a person who leaves A increase public net expenditures by $a-t$.

First order conditions way then be rewritten as

$$
\begin{aligned}
u'(2b)2B &= \lambda\left[2(b+t)\frac{\partial W}{\partial b}+(2b-(a-t))\frac{\partial A}{\partial b}-2B\right] \\
\overline{u'^{A}}A &= \lambda\left[(a+t)\frac{\partial W}{\partial a}-\left(2b-(a-t)\right)\frac{\partial B}{\partial a}-A\right]
\end{aligned}
$$

(15) or

$$u'(2b) = \lambda\left[\frac{2}{2B}(b+t)\frac{\partial W}{\partial b} + \frac{2b-(a-t)}{2B}\frac{\partial A}{\partial b} - 1\right]$$

$$\overline{u'^{A}} = \lambda\left[\frac{a+t}{A}\frac{\partial W}{\partial a} - \frac{2b-(a-t)}{A}\frac{\partial B}{\partial a} - 1\right]$$

Since individuals choose for themselves, $(w,r) \in \mathcal{A} \Rightarrow a - g + w + r - t - d > 2b$ and hence $\overline{u'^{A}} < u'(2b)$.[3] Hence, a necessary condition for a solution to (15) to exist is

(16) $$\left|\frac{2}{2B}(b+t)\frac{\partial W}{\partial b} + \frac{2b-(a-t)}{2B}\frac{\partial A}{\partial b}\right| > \left|\frac{a+t}{A}\frac{\partial W}{\partial a} - \frac{2b-(a-t)}{A}\frac{\partial B}{\partial a}\right|$$

This will be the case if the labour supply effects of the education programme relative to the passive benefit are not too large, and/or if $2b-(a-t) > 0$ is combined with a large programme shifting effect (large values of $\frac{\partial B}{\partial a}$ and $\frac{\partial A}{\partial b}$). The inequality $2b-(a-t) > 0$ means that the public budget will be improved for each individual who moves from the passive benefit to the education programme. If (16) is fulfilled, the government choose a to be large relative to b. On the other hand, (15) are

[3] Note that a solution to a problem where the government have full information about individuals' characteristics and decides who is going to be in which state has first order conditions as (15) but with $\frac{\partial W}{\partial b} = \frac{\partial A}{\partial b} = \frac{\partial W}{\partial a} = \frac{\partial B}{\partial a} = 0$, so that marginal utilities are equalised across states. This means that $a - g + w + r - t - d > 2b$ is true for some individuals (some $(w,r) \in \mathcal{A}$), while the reverse is true for others.

written under the assumption $b > a - g$. Therefore, is appears logic to consider the alternative case.

The case $a - g > b$ (figure 2 and 4)

It is immediate that $\frac{\partial B}{\partial a} = 0, \frac{\partial W}{\partial b} = 0$. Proceeding as above, we get

$$
\begin{aligned}
u'(b+a-g)B &= \lambda\left[t\frac{\partial W}{\partial b}2 - \frac{\partial A}{\partial b}a + \frac{\partial A}{\partial b}t - (a+b)\frac{\partial B}{\partial b} - B \right] \\
&= \lambda\left[(2t+a+b)\frac{\partial W}{\partial b} - (a-t+a+b)\frac{\partial A}{\partial b} - B \right] \\
&= \lambda\left[(b+t)\frac{\partial A}{\partial b} - B \right] \\
u'(b+a-g)B + \overline{u'^{A}}A &= \lambda\left[(2t+a-t)\frac{\partial W}{\partial a} - (a+b-(a-t))\frac{\partial B}{\partial a} - A - B \right] \\
&= \lambda\left[(t+a)\frac{\partial W}{\partial a} - (a+t)\frac{\partial B}{\partial a} - A - B \right] \\
&= \lambda\left[(t+a)\frac{\partial W}{\partial a} - A - B \right]
\end{aligned} \tag{17}
$$

or

$$
\begin{aligned}
u'(b+a-g) &= \lambda\left[\frac{(b+t)}{B}\frac{\partial A}{\partial b} - 1 \right] \\
\frac{u'(b+a-g)B + \overline{u'^{A}}A}{A+B} &= \lambda\left[\frac{t+a}{A+B}\frac{\partial W}{\partial a} - 1 \right]
\end{aligned} \tag{18}
$$

In particular it is interesting to discuss whether $b = 0$ can be a solution to (18) – an interpretation of this is that the policy much used in the real world with programme-participation as a condition for claiming benefits have a foundation in theory on optimal social policy. As can be seen, it cannot be precluded that equation (18) might be fulfilled for $b = 0$. This may happen if the programme shifting effect ($\frac{\partial A}{\partial b}$) is large and the labour supply effect ($\frac{\partial W}{\partial a}$) is small. If $b = 0$, individuals on passive benefits lives for both periods of the education benefit a. Hence for $b = 0$ to be part of a solution, the education benefit cannot be to low and/or the marginal utility cannot increase to much as its argument decreases. Note that in the model above, the utility function u is a function of two-period income. Alternatively we could assume that individuals consume current income and base choice of state with a (concave) current utility function v such that state $\mathcal{B}$ is preferred to $\mathcal{W}$ by the individual if $v(b) + v(a - g) > v(w - t - d) + v(w - t - d)$. An individual in $\mathcal{B}$ would contribute to government utility by $u(v(b) + v(a - g))$. In this case the much-used assumption $v'(b) \to \infty$ as $b \to 0$ would preclude $b = 0$ as part of optimal policy. See section 6.

The analysis above presumes (12) is fulfilled, i.e. that $B, A > 0$ is always true. If this is not the case, we have to investigate candidates for a solution to the government's problem in various sets of a, b, t defined by $B, A > 0$; $B = 0, A > 0$; $B > 0, A = 0$; and $B = A = 0$ respectively. The welfare levels for these solutions have to be compared. If, for example, $B, A > 0$ for some values of a, b, t but B converges quickly to zeros as b decreases to some positive value, the right hand side of the first equation in (18) raises to infinity because the 'relative' programme shifting effect, $\frac{\partial A / \partial b}{B}$, increases without bounds.

Consequently, if all individuals leave state $\mathcal{B}$ before b reaches some lower level, $\underline{b}$, it might be optimal only to open the education programme, that is, to condition social benefits on programme participation.

4. Private education programmes

In this section we consider the case where participation in ‘private’ education programmes can take place without the government’s knowledge. Such private education programmes improve productivity and imply disutility in the same way as ‘public’ programmes. The only difference is that the government can observe whether an individual participate in a public programme. The education benefit rate a is the rate an individual gets if he participates in a public programme. The main effect of the introduction of private programmes is that $a > b$ is required for anyone to participate in the public programmes. If $a \leq b$, an individual who wants to improve productivity will claim benefits b and participate in a private education programme.

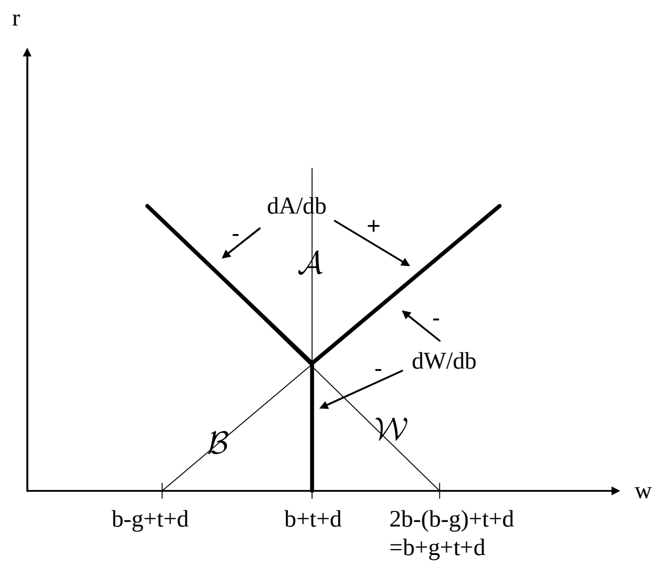
To consider candidates for optimal policy, consider first the case $a < b$, so the government only has to decide the size of the passive benefit rate b.

The set $\mathcal{A}$ now denotes individuals who participate in private education in the first period and work in the second. Participants obtain income $b - g + w + r - t - d$. Partial derivatives of measures of sets have the following sign, $\frac{\partial W}{\partial a}=\frac{\partial A}{\partial a}=\frac{\partial B}{\partial a}=0$ and

$\frac{\partial W}{\partial b}<0, \frac{\partial B}{\partial b}>0, \frac{\partial A}{\partial b}<0$ or $\frac{\partial A}{\partial b}>0$.

Figures 1 and 2 are replaced by figure 5.

Figure 5. Sets with private education and $a<b$



The government's welfare function is still

$$V = u(2b)B + \overline{u^A}A + \overline{u^W}W$$

where

(19) $$\overline{u^A} = \frac{1}{A}\int_{(w,r)\in\mathcal{A}} u(b-g+w+r-t-d)\, f(w,r)\mathrm{d}(w,r)$$

and also

$$\overline{u'^A} = \frac{1}{A}\int_{(w,r)\in\mathcal{A}} u'(b-g+w+r-t-d)\, f(w,r)\mathrm{d}(w,r)$$

and the budget is

(20) $$F = 2tW - (b-t)A - 2bB = 0$$

We get the first order condition

(21) $$\begin{aligned}\frac{\partial V}{\partial b} &= 2u'(b)B + \overline{u'^A}A - \lambda\left[2t\frac{\partial W}{\partial b} - (b-t)\frac{\partial A}{\partial b} - 2b\frac{\partial B}{\partial b} - 2B - A\right] \\ &= 2u'(b)B + \overline{u'^A}A - \lambda\left[(2t+2b)\frac{\partial W}{\partial b} + (2b-(b-t))\frac{\partial A}{\partial b} - 2B - A\right] \\ &= 2u'(b)B + \overline{u'^A}A - \lambda\left[2(b+t)\frac{\partial W}{\partial b} + (b+t)\frac{\partial A}{\partial b} - 2B - A\right] = 0\end{aligned}$$

The individuals moving from work to passive benefit increase net public expenditures by $2(b+t)\left|\frac{\partial W}{\partial b}\right|$. The term $(b+t)\frac{\partial A}{\partial b}$ is explained by two types of transitions. First, if an individual leaves work for education rather than passive benefits, the contribution to public net expenditures will be $(b+t)$ *less* for each of these individuals. Second, if an individual transits from education to passive benefits in response to the increase in b, public net expenditures increase by $(b+t)$.

The candidates to a solution to the government's problem are thus: solutions to (21), solutions to (15) with $a > b$ (where $b > a - g$ is assumed), solutions to (16) where $a > b$ is fulfilled because $b < a - g$ is presumed. To find the optimal social policy, the welfare level of these candidates should be compared.

5. Credit constraints

In this section we assume that in each period each individual consumes his current income. Two-period utility is $v(y_1) + v(y_2)$ where y_1, y_2 are income in period 1 and 2 (adjusted for disutility) and v is concave.

Individuals' choices of actions are written as in equation (1) to (6') modified with the use of v. For example: action WW is chosen if the following conditions are satisfied

(1'') $\quad WW \succ BB \Leftrightarrow v(w-t-d) + v(w-t-d) > v(b) + v(b)$

(2'') $\quad WW \succ AW \Leftrightarrow v(w-t-d) + v(w-t-d) > v(a-g) + v(w+r-t-d)$

(3'') $\quad WW \succ BA \Leftrightarrow v(w-t-d) + v(w-t-d) > v(b) + v(a-g)$

If the total two-period disutility-adjusted incomes for an individual are the same in two states, the individual is indifferent when she maximize total income (as in section 2) but with consumption equal to income, she will prefer the state with the more equal income across the two periods. In figures 6 and 7, we show how figures 1 and 2 are changed when consumption is set equal to current income.

Figure 6. States and credit constraints (case $b > a - g$)

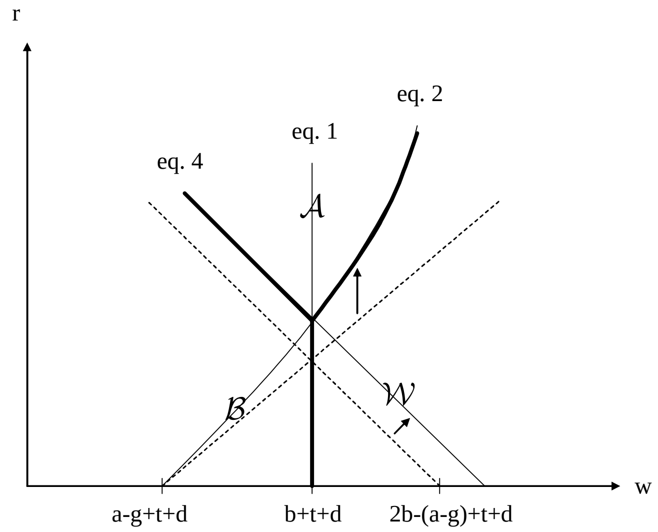
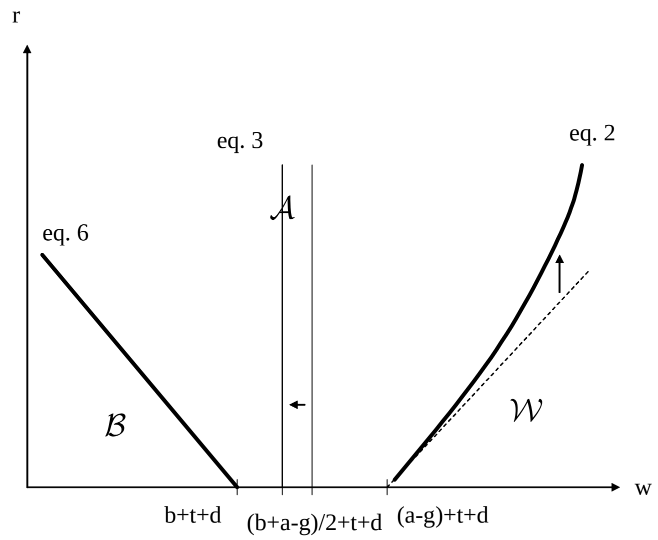


Figure 7. States and credit constraints (case $b < a - g$)



Each individual contributes by $u(v(y_1)+v(y_2))$ to the government's welfare function, V.

Marginal utilities are modified as

(22)
$$\begin{aligned}\overline{u'^A} &= \frac{1}{A}\int_{(w,r)\in\mathcal{A}} v'(a-g)u'(v(a-g)+v(w+r-t-d)\,f(w,r)\mathrm{d}(w,r) \\ &= v'(a-g)\frac{1}{A}\int_{(w,r)\in\mathcal{A}} u'(v(a-g)+v(w+r-t-d)\,f(w,r)\mathrm{d}(w,r)\end{aligned}$$

and, in case $b > a-g$,

(23)
$$u'(2v(b)) = 2v'(b)u'(v(b))$$

and, in case $b < a-g$,

(24)
$$\begin{aligned}u'_b(v(b)+v(a-g)) &= v'(b)u'(v(b)+v(a-g)) \\ u'_a(v(b)+v(a-g)) &= v'(a-g)u'(v(b)+v(a-g))\end{aligned}$$

First order conditions analogous to (15) and (16) now become, in case $b > a-g$,

(25)
$$\begin{aligned}2v'(b)u'(2v(b)) &= \lambda\left[\frac{2}{2B}(b+t)\frac{\partial W}{\partial b}+\frac{2b-(a-t)}{2B}\frac{\partial A}{\partial b}-1\right] \\ v'(a-g)\overline{u'^A} &= \lambda\left[\frac{a+t}{A}\frac{\partial W}{\partial a}-\frac{2b-(a-t)}{A}\frac{\partial B}{\partial a}-1\right]\end{aligned}$$

Case $b < a-g$

(26)
$$\begin{aligned}v'(b)u'(v(b)+v(a-g)) &= \lambda\left[\frac{(b+t)}{B}\frac{\partial A}{\partial b}-1\right] \\ \frac{v'(a-g)\left[u'(v(b)+v(a-g))B+\overline{u'^A}\right]A}{A+B} &= \lambda\left[\frac{t+a}{A+B}\frac{\partial W}{\partial a}-1\right]\end{aligned}$$

If the much-used condition $v'(y)\to\infty$ as $y\to 0$ is assumed, then neither $b=0$ or $a=g$ can be part of an optimal social policy.

6. Conclusion

The paper analyses the principles for the size of the passive benefit rate compared to the benefit rate in education programmes and whether these benefit rates could be zero in an optimal social policy. A generous education benefit is good because it stimulate labour supply from the otherwise unemployed participants, but bad because it stimulates over-invest in education from otherwise employed individuals. The optimal size of the education benefit rate depends on the distribution of characteristics, i.e. whether there are relatively many otherwise unemployed or employed who will participate. Zero benefit rates are under reasonable trivially shown not to be part of optimal social policy.

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